Econometric Reduction Theory and Philosophy

Genaro Sucarrat
The Economics Department
Universidad Carlos III de Madrid

Econometric reduction theory provides a comprehensive probabilistic framework for the analysis and classification of the reductions (simplifications) associated with economic models. However, the available approaches to econometric reduction theory are unable to satisfactory accommodate a commonplace theory of social reality, namely that the course of history is indeterministic, that history does not repeat itself and that the future depends on the past. Using concepts from philosophy a solution to these shortcomings is proposed, which in addition permits new reductions, interpretations and definitions.

Econometric reduction theory

Economic models are simplifications of reality. But how simple are they? And what exactly is the nature of their simplifications? Econometric reduction theory gives an answer to these questions. It is the study, in terms of probability concepts, of the simplifications implicit or explicit in theoretical and empirical economic models, and a reduction is simply the probability structure that results from a simplification (broadly defined).

The starting point of econometric reduction theory can therefore be viewed as the foundation of all quantitative economic analysis, since all theoretical and empirical models—at least conceptually—can be obtained via sequential simplifications starting from the initial structure.

David F. Hendry's reduction theory

The most well-known econometric reduction theory is that of David F. Hendry (1995, chapter 9), which distinguishes between twelve stages of reductions. The number of simplifications is not an objectively given number, since one may always consider a finer sequence of simplifications. Also, the order of the simplifications is sometimes a matter of choice. So many of the differences between reduction theories are simply due to a different emphasis among scholars. The fame of Hendry's theory is partly due to its generality (its starting point can be viewed as comprising or being equivalent to the starting points of the other theories), and partly because it is commonly used to justify the widespread General-to-Specific (GETS) approach to econometric modelling. GETS modelling mimicks reduction theory, since it starts with a general model and then simplifies sequentially while checking the statistical adequacy of each simplification.

The starting point of Hendry's theory is a finite sequence of random theory variables $\mathbf{U}^*$ equal to $\{\mathbf{U}^*_1,...,\mathbf{U}^*_t,...,\mathbf{U}^*_T\}$, where each vector of theory variables $\mathbf{U}^*_t$ is equal to $\{U^*_1,...,U^*_i,...,U^*_{I(t)}\}$. The notation $I(t)$ means the number of theory variables can vary with t. The theory variables $\mathbf{U}^*$ are defined on the probability space $(\Omega, \mathcal{F}, P)$, where Ω is the outcome-set, $\mathcal{F}$ is the event-set (a σ-algebra made up of subsets of Ω) and P is the probability function. Together, the theory variables defined on the probability space constitute the "economic mechanism" under study. That is, the starting point of Hendry's reduction theory. Now, one may ask: How we can be sure a set of conjectured theory variables actually exist? We cannot. Indeed, the postulation of theory variables can in itself be viewed as a reduction. I will return to this question towards the end. The first simplification in Hendry's theory leads to the Data Generating Process (DGP), which is the result of collecting data in an attempt to measure the theory variables. The data may be entirely unrelated to the theory variables they purport to measure, so already after the first reduction one may be far off the intended target. Next, Hendry outlines eleven additional reductions: (2) Data-transformation and aggregation, (3) specification of the parameters of interest, (4) data-partitioning, (5) marginalisation, (6) sequential factorisation, (7) the mapping to covariance-stationarity, (8) conditional factorisation, (9) the assumption of constancy, (10) lag truncation, (11) functional form approximation and (12) the final model. Philosophical issues enter primarily in relation to the first stage, so the subsequent stages will not be our focus here.

A major shortcoming of reduction theory, and a solution

Econometric reduction theory constitutes a powerful and very general framework for the study, analysis and classification of the simplifications associated with economic models. However, the available approaches are unable to satisfactory accommodate a commonplace theory of social reality, namely that the human world is made up of indeterministic, historically inherited particulars. Or, in lay-man terms, that the course of history is indeterministic (indeterminism), that history does not repeat itself (particularism), and that the future depends on the past (historical inheritance). This is a major shortcoming of reduction theory, since it means many simplifications cannot even be stated in terms of probabilistic language. The main source of the shortcoming is that the currently available reduction theories are silent or too imprecise and incomplete about the nature of the underlying outcome space. In *Econometric Reduction Theory and Philosophy*[1] I propose a solution to this shortcoming. Specifically, I propose that the elements of the outcome-set Ω, that is, the outcomes ω, should be devised as worlds with a certain structure. The idea of a world or possible world is used extensively in modern philosophy for a variety of purposes, and I simply propose that the outcome-set should be viewed as the set of possible worlds. The structure of the worlds I propose is unrestrictive, it bridges econometric reduction theory and philosophy (metaphysics in particular), it provides a solution to several shortcomings in econometric reduction theory, and it enables many new reductions, concepts and definitions. I will return to this later, but first I state the exact structure I propose:

Definition: Outcome-set consisting of indeterministic worlds made up of historically inherited particulars. Let $(\Omega, \mathcal{F}, P)$ be a probability space and let each $\omega \in \Omega$ be equal to a non-stochastic continuous time process $\{s(t) : t \in [0,\infty)\}$, where the $s(t)$ are referred to as the worldly states-of-affairs at t. The outcome space Ω is said to consist of possible worlds made up of indeterministic and historically inherited particulars if:

a) There exists more than one world ω in Ω and at least two unequal worlds $\omega; \omega' \in \Omega$ intersect: $\omega \cap \omega' \neq \varnothing$ (indeterminism).

b) For each $\omega \in \Omega$: For all pairs $t, t' \in [0,\infty)$ such that $t \neq t'$ and $s(t') \in \omega$, then $s(t) \neq s(t')$ (particularism)

c) For each pair of unequal worlds $\omega^1, \omega^2 \in \Omega$, that is, $\omega^1 \neq \omega^2$: If $\omega_t^1 \neq \omega_t^2$, then $s^1(t') \neq s^2(t'')$ for all $t', t'' \in (t, \infty)$ where $s^1(t') \in \omega^1$ and $s^2(t'') \in \omega^2$. (historical inheritance)

Crudely, the first property a) states that the course of history is indeterministic, the second property b) states that history does not repeat itself, and the third property c) states that the future depends on the past. The interested reader is referred to the paper for a more detailed explication and motivation of the definition, and table 1 contains a summary of what could be a revised version of Hendry' reduction theory.

Interpreting the ω as worlds is not restrictive and retains the intuitive use of probability algebra. For example, if we would like to say that $A \in \mathcal{F}$ denotes the event that (say) 10% of the labour force of an economy is unemployed at t, then the only change in interpreting the ω as a world is that A now denotes the set of all worlds in which 10% of the labour force of a certain economy is unemployed at t. More formally, $A = \{\omega : 10\% \text{ unemployed at } t\}$. If the worlds are bounded backwards, then the interpretation becomes that A denotes the set of all worlds in which 10% of an economy is unemployed at t given the history of the world up to $t = 0$. Another common practice is to interpret the outcome set Ω as a set of possible "states-of-

Table 1. Starting points, actions and resulting reductions involved in a modified version of the first stage of Hendry's reduction theory

Reduction no.	Starting point and resulting reduction	Action	Simplification/information loss
	A probability space $(\Omega, \mathcal{F}, P)$ where the outcome-space Ω consists of indeterministic worlds made up of historically inherited particulars		
		The delineation and definition of theory variables $\mathbf{U}^*$	The events that are excluded from analysis together with their associated probabilities: $\mathcal{F} - \mathcal{F}^*$ and $P - P^*$
1.	The economic mechanism: The theory variables $\mathbf{U}^* = (\mathbf{U}_1^*, \ldots, \mathbf{U}_T^*)$ together with $(\Omega, \mathcal{F}^*, P^*)$, where $\mathcal{F}^* \subset \mathcal{F}$ and $P^* \subset P$		
		Data collection and recording of $\mathbf{U}_t \in \mathbf{U}$, that is, the process of trying to measure the $\mathbf{U}_t^* \in \mathbf{U}^*$ variables	Functions of the discrepancy between $\mathcal{F}^*$ and $\mathcal{F}^D$, and the discrepancy between P^* and P^D. For example, $\mathcal{F}^* - \mathcal{F}^D$ and $P[\bigcup_{n=1}^{\infty}(\mathcal{F}^* - \mathcal{F}^D)]$, or alternatively $\mathcal{F}^D - \mathcal{F}^*$ and $P[\bigcup_{n=1}^{\infty}(\mathcal{F}^D - \mathcal{F}^*)]$
2.	The data generation process (DGP): The data variables $\mathbf{U} = (\mathbf{U}_1, \ldots, \mathbf{U}_T)$ together with $(\Omega, \mathcal{F}^D, P^D)$, where $\mathcal{F}^D \subset \mathcal{F}$ and $P^D \subset P$		
		Approximating the conditional probability function of the DGP, given by $P^D_{\mathcal{C}^E}$, by means of an estimation and inference model P^E	The restriction of limiting the analysis to the set $\mathcal{C}^E = \{C_1^E, C_2^E, \ldots\}$ where P^E is approximately equal to $P^D_{\mathcal{C}^E}$, and the discrepancy between $P^D_{\mathcal{C}^E}$ and P^E
3.	The estimation and inference model P^E		
		Restricting the estimation and inference model P^E to be derivable from the theory model P^T	The restriction of limiting the analysis to the set $\mathcal{C}^T = \{C_1^T, C_2^T, \ldots\}$ where P^T is approximately equal to $P^D_{\mathcal{C}^T}$, the discrepancy between $P^D_{\mathcal{C}^T}$ and P^T, and the loss associated with $\bigcup_{n=1}^{\infty} C_n^E - \bigcup_{m=1}^{\infty} C_m^T$ and $P(\bigcup_{n=1}^{\infty} C_n^E - \bigcup_{m=1}^{\infty} C_m^T)$
4.	The theory model P^T together with the implied estimation and inference model P^E		

affairs" or " facts". In possible worlds terminology a state-of-affairs or fact at *t* is now the set of all worlds in which a certain state-of-affairs or fact attains at *t*. Finally, the possible worlds interpretation also accommodates more complex types of events (and their combinations), including socalled "interval" events. For example, we may want to specify an event A equal to the set of worlds in which 10% of the labour force of an economy is registered as unemployed over the time interval, say, $[t_0; t_1]$ with $t_0 < t_1$. Or, $A = \{\omega : 10\%$ unemployed during $[t_0; t_1]\}$.

Reduction theory revisited

Although reduction theory provides a comprehensive framework for the analysis of the relation between social reality and econometric models thereof, it nevertheless has several shortcomings. The first shortcoming was pointed to above, and effectively means that the current approaches to reduction theory are unable to satisfactorily reconcile two conflicting views on social reality. The first view is the commonplace theory of social reality that the course of history is indeterministic, that history does not repeat itself, and that the future depends on the past. The second view is that there are stable laws or regularities regarding the relationship between variables, an idea which underlies most econometric practice. For example, in Hendry's theory the economic mechanism under study is a regularity-entity that *can* change over time. In other words, periods of no-change means the regularities of the economic mechanism are not changing. According to the commonplace theory of social reality, however, there is no *a priori* reason for stable or enduring regularities to exist, so their existence is an empirical question. Conceptually this is not necessarily incompatible with Hendry's theory. But since neither Hendry nor the other approaches to econometric reduction theory give a probabilistic account of why and how the economic mechanism changes, they are unable to provide probabilistic reduction analysis with reference to the same initial probability space. A solution is simply to specify the outcome set as consisting of indeterministic worlds made up of historically inherited particulars. This means reduction analysis can be undertaken with reference to the same initial probability space throughout all reductions, and the (conditional) existence of regularities — either across time and/or space — can be obtained as (conditional) reductions.

A second shortcoming of the current approaches to reduction theory is that reduction analysis of the relation between continuous and discrete time models is not straightforward. Indeed, conceptually, the standard approach is internally contradictive. With the proposed structure on the underlying outcome space, however, reduction analysis on the relation between continuous and discrete time models is readily enabled without internal conceptual contradictions. In fact, the relation between events of a wide range of additional temporal structures can be analysed, including intervals, processes, and overlapping intervals and processes.

A third shortcoming of the current approaches to econometric reduction theory is the (implicit or explicit) view that there objectively exists a "complete set" of theory variables "relevant to the economy under investigation" (Hendry 1995, p. 345). If the course of history is indeterministic, if history does not repeat itself and if the future depends on the past, the number of theory variables of objective relevance for any economic event is enormous, maybe even infinite, bounded by imagination only. In the words of David Lewis:

"Any particular event that we might wish to explain stands at the end of a long and complicated causal history...We have the icy road, the bald tire, the drunk driver, the blind corner, the approaching car, and more. Together, these cause the crash. Jointly they suffice to make the crash inevitable, or at least highly probable, or at least much more probable than it would otherwise have been... But these are by no means all the causes of the crash. For one thing, each of these causes in turn has its causes; and those too are causes of the crash. So in turn are their causes, and so, perhaps, *ad infinitum*." — Lewis (1986, p.214)

In practice, any economic investigation can focus attention on only a (relatively small) finite number of variables that may be of relevance for the purpose of the analysis. Specifying the outcome set as consisting of indeterministic worlds made up of historically inherited particulars enables us to treat the formulation or choice of theory variables as a simplification or the perspective from which we study an issue, an idea which in economics is associated with (amongst others) Max Weber, Gunnar Myrdal and Joseph Schumpeter.

A fourth shortcoming in Hendry's theory is that the underlying probability space is transformed when data are collected. The theory is therefore unable to provide probabilistic reduction analysis with reference to the same initial probability space of the relation between the theory and data variables. The suggested structure of the fundamental outcome set means the initial probability space does not change, and enables a probabilistic definition of the absence of data measurement error.

Finally, traditional approaches to historical conditioning in probability and statistics do not adequately accommodate the uniqueness and dependence of historical context. Indeed, the traditional notions of history are "too large" in the sense that they contain too many worlds. The proposed structure of the outcome space enables a more correct definition of history in terms of probability concepts.

Notes

[1] The latest version of this article is available via my webpage http://www.eco.uc3m.es/sucarrat/index.html. An earlier version is available as Universidad Carlos III de Madrid Working Paper 09-10 in the Economic Series: http://e-archivo.uc3m.es/dspace/bitstream/10016/3773/1/we091005.pdf.

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