BAYESIAN ECONOMETRICS: Tutorial 3 (VAR)
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Exercise 1
This exercise illustrates Bayesian inference for unrestricted VAR models.

Consider a bivariate VAR for two interest rates (in levels), a short rate (r_t) and a long rate (R_t).[1] Include 6 lags and a constant term in the VAR, so that in each equation you have 6 lags of r_t and 6 lags of R_t, plus the constant term. Thus the matrix B of the VAR written as $Y = ZB + E$ (slide 153) has $k = 13$ and $n = 2$ columns. Let us write the VAR explicitly like this for date t:

$$\begin{aligned} r_t &= c_1 + \sum_{i=1}^{6} \beta_{1,i} r_{t-i} + \sum_{i=7}^{12} \beta_{1,i} R_{t-i} + \epsilon_{1,t} && (1) \\ R_t &= c_2 + \sum_{i=1}^{6} \beta_{2,i} r_{t-i} + \sum_{i=7}^{12} \beta_{2,i} R_{t-i} + \epsilon_{2,t} && (2) \end{aligned}$$

The data are monthly, covering the period 1960.1 until 1996.12 (444 observations). They are available in the file FranceInterestRates.txt. This file contains two columns of data: the series r_t in the first column and the series R_t in the second column.

For each algorithm below, you are asked to compute several posterior results:
(pr1) The posterior means and standard deviations of the elements of B;
(pr2) The posterior means, standard deviations and marginal densities of $\sum_{i=1}^{6} \beta_{1,i}$, $\sum_{i=7}^{12} \beta_{1,i}$, $\sum_{i=1}^{6} \beta_{2,i}$, and $\sum_{i=7}^{12} \beta_{2,i}$;
(pr3) The posterior means and standard deviations and marginal densities of σ_1, σ_2, and $\rho = \sigma_{12}/(\sigma_1 \sigma_2)$, where $\sigma_i = \sqrt{\sigma_{ii}}$ $(i = 1, 2)$ and σ_{ij} is an element of the Σ matrix;
(pr4) The posterior means, standard deviations and marginal densities of the smallest eigenvalue of the matrix A'A where $A = I_n - \sum_{j=1}^{6} A_i$ when the VAR is written as

$$(I_n - \sum_{i=1}^{6} A_i L^i) x_t = c + \epsilon_t,$$

where $x_t = (r_t \ R_t)'$, $c = (c_1 \ c_2)'$, $\epsilon_t = (\epsilon_{1,t} \ \epsilon_{2,t})'$, and $A_i = \begin{pmatrix} \beta_{1,i} & \beta_{1,i+6} \\ \beta_{2,i} & \beta_{2,i+6} \end{pmatrix}$ for $i = 1, 2, \ldots, 6$.[2]

[1] The short rate is the call money rate (for overnight lending between banks) and the long rate is a rate on ten-year public and semi-public bonds.

[2] The VAR has a unit root if $|A| = 0$. If this the case $A'A$ has also a zero eigenvalue. We check the eigenvalues of $A'A$ because they are real (those of A my be complex). By computing

Program the following computations:

1. Using a non-informative prior, compute (pr1)-(pr4) by direct sampling (slide 163), and by Gibbs sampling (slide 164). Compute also (pr1) analytically (see slide 162, posterior of B in formula (13)) and compare with the results computed by the two sampling algorithms.

2. Using a Minnesota prior, compute (pr1)-(pr4) by Gibbs sampling (slide 171). For the Minnesota prior, fix the σ_i values equal to the posterior means estimated with a NIP prior, and set $\lambda = 0.2$ and $\theta = 0.5$ (or to any other values that you consider reasonable).

NB: for results based on MCMC algorithms, do the necessary convergence checks (tests and graphs) for the posterior means.

Exercise 2
This exercise illustrates Bayesian inference for a VAR model subject to zero restrictions. Refer to section 3.5 of the course.

In the bivariate VAR of the previous exercise, impose the restrictions $\beta_{2,i} = 0$ for $i = 1, 2, ..., 6$ (non causality of r_t for R_t). Compute (pr1)-(pr4) by Gibbs sampling assuming a NIP.

Optionally: assume a Normal prior for β (us the idea of the Minnesota prior to design it practically) and a diffuse prior for Σ. Write the conditional posterior densities $\beta|\Sigma, d$ (Normal) and $\Sigma|\beta, d$ (IW) and compute (pr1)-(pr4) by Gibbs sampling .

the posterior of the smallest eigenvalue we can see if it has a lot of probability mass close to 0. If that is the case, this indicates that A may have a reduced rank, and that the VAR system may be subject to cointegration restrictions.