Matlab Tutorials

April 2013

Tutorial 2 on Matlab

Solutions are provided in the directory Tutorial2Solutions :

- Question 1 : Gibbs_regression.m
- Question 2 : MH_regression.m

Running a Matlab function :

- Click on the m file that you want to run
- Copy and paste in the 'command window' the first line without the word 'function'
- Choose adequate inputs for the function.
- For example (see Gibbs_regression.m) :

  [Simu] = Gibbs_regression(y,X,10000,1,1)

Tutorial 2 on Matlab

Regression :

$$\Delta \log(W_t/P_t) = \alpha_0 + \beta_1 \Delta \log P_t + \beta_2 \Delta \log Q_{t-1} + \beta_3 UR_t + \epsilon_t$$

The data are available in the mat file data.mat

- Annual observations for Belgium, covering the period 1954-1976.
- The mat file contains

$$y = \log(W_t/P_t) \text{ and } X = [1 \;\; \Delta \log P_t \;\; \Delta \log Q_{t-1} \;\; UR_t]$$

- Parameter set : $\beta = (\alpha_0, \beta_1, \beta_2, \beta_3)'$ and σ^2
- Recall that if $Z \sim G_2(a, b)$ then $Z \sim G(\frac{a}{2}, \frac{b}{2})$

Posterior Distribution

$$f(\beta, \sigma^2 | Y, X) \propto f(Y|\beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Four different priors :

1. Diffuse prior (Direct Sampling is possible)

$$f(\beta, \sigma^2) \propto \frac{1}{\sigma^2}$$

2. Conjugate prior but partially non-informative (Direct Sampling possible)

$$f(\beta, \sigma^2) \propto f(\beta_1) f(\sigma^2)$$

   where $\beta_1 | \sigma^2 \sim N(\beta_0, M_0^{-1}\sigma^2)$ and $\sigma^2 \sim IG(v_0, s_0)$

3. Normal prior for β_1 and diffuse for the other parameters

$$f(\beta, \sigma^2) \propto f(\beta_1) \frac{1}{\sigma^2}$$

   where $\beta_1 \sim N(\beta_0, M_0^{-1})$

4. Beta prior for β_1 and diffuse for the others.

$$f(\beta, \sigma^2) \propto f(\beta_1) \frac{1}{\sigma^2}$$

   where $\beta_1 \sim$ symmetric Beta distribution on $[-0.3, 0.3]$

Posterior Distribution

$$f(\beta, \sigma^2 | Y, X) \propto f(Y|\beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

[Simu] = Gibbs_regression(y,X,nb_MCMC,prior_choice,graph)

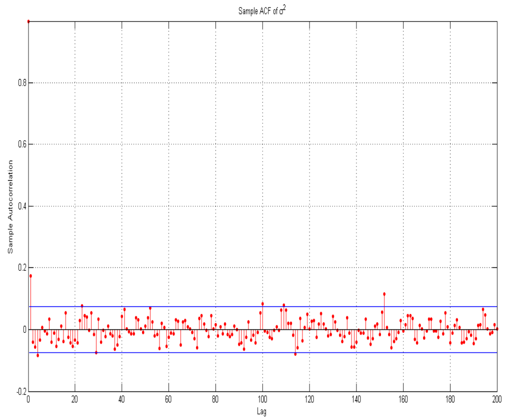
1. y and X : dependent and explanatory variables.
2. nb_MCMC : number of MCMC iterations.
3. prior_choice : from 1 to 3
   provides posterior distribution under prior 1 to 3

   The last prior (Beta distribution) is dealt in MH_regression.m
4. graph : = 1 displays some convergence graphics.

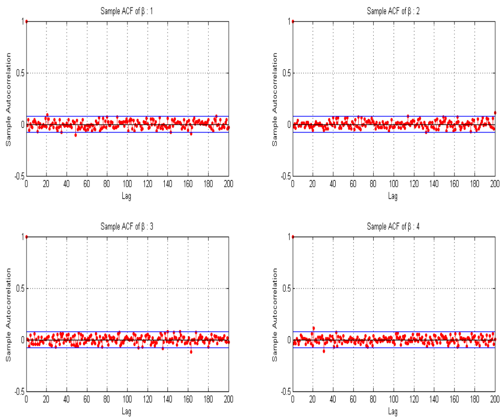
To run the function :

[Simu] = Gibbs_regression(y,X,10000,1,1)

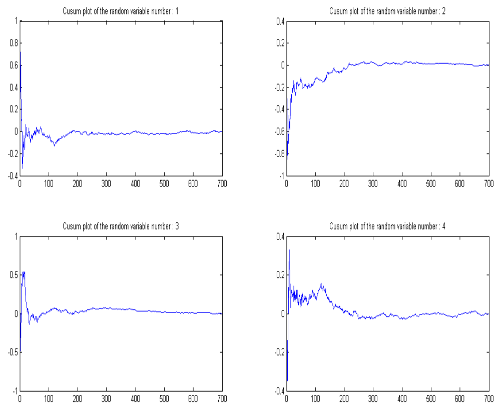
[Simu] = Gibbs_regression(y,X,10000,1,1)



ACF σ^2



ACF β



Cusum plot β

A structure Simu

```
Simu =

                      DS_sigma: [1000x1 double]
                       DS_beta: [4x1000 double]
                     post_beta: [4x700 double]
                    post_sigma: [700x1 double]
                      beta_ols: [4x1 double]
                     sigma_ols: 0.00
                post_mean_beta: [4x1 double]
                 post_std_beta: [4x1 double]
post_var_beta_rao_blackwellisation: [4x4 double]
                 post_var_beta: [4x4 double]
               post_mean_sigma: [700x1 double]
                post_std_sigma: [700x1 double]
                   test_Geweke: [4x1 double]
                         cusum: [1x1 struct]
```

Structure

A structure Simu

```
Simu =

                               DS_sigma: [1000x1 double]
                                DS_beta: [4x1000 double]
                              post_beta: [4x700 double]
                             post_sigma: [700x1 double]
                               beta_ols: [4x1 double]
                              sigma_ols: 0.00
                         post_mean_beta: [4x1 double]
                          post_std_beta: [4x1 double]
    post_var_beta_rao_blackwellisation: [4x4 double]
                          post_var_beta: [4x4 double]
                        post_mean_sigma: [700x1 double]
                         post_std_sigma: [700x1 double]
                            test_Geweke: [4x1 double]
                                  cusum: [1x1 struct]
```

To get one field of the structure : In the command window

Simu.post_mean_beta
Simu.post_std_beta
Simu.post_mean_sigma
Simu.post_std_sigma

Posterior Distribution

$$f(\beta, \sigma^2|Y, X) \propto f(Y|\beta, \sigma^2, X) f(\beta, \sigma^2|X)$$

Diffuse prior :

1. Direct Sampling :

$$f(\beta, \sigma^2|Y, X) = f(\beta|Y, \sigma^2, X) f(\sigma^2|Y, X)$$

where

$$\beta|Y, \sigma^2, X \sim N(\beta_{\text{ols}}, \sigma^2 (X'X)^{-1})$$

$$\sigma^2|Y, X \sim IG(\frac{T-k}{2}, \frac{(Y - X\beta_{\text{ols}})'(Y - X\beta_{\text{ols}})}{2})$$

```
for i=1:nb_MCMC
    sigma = 1/gamrnd((T-dimension)/2,2/SSR);
    Simu.DS_beta(:,i) = mvnrnd(beta_ols,sigma*inv_X_X)';
    Simu.DS_sigma(i) = sigma;
end
```

Posterior Distribution

$$f(\beta, \sigma^2 | Y, X) \propto f(Y | \beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Diffuse prior :

- Gibbs sampling
  1. $\beta | Y, \sigma^2, X \sim N(\beta_{\text{ols}}, (X'X)^{-1}\sigma^2)$
  2. $\sigma^2 | Y, \beta, X \sim IG(\frac{T+k}{2}, \frac{\sum_{t=1}^{T} \epsilon_t^2}{2})$

Conjugate prior : $\beta_1 | \sigma^2 \sim N(\beta_0, m_0^{-1}\sigma^2)$ and $\sigma^2 \sim IG(v_0, s_0)$

- Gibbs sampling
  1. $\beta | Y, \sigma^2, X \sim N((X'X + M_0)^{-1}(X'y + M_0 B_0), (X'X + M_0)^{-1}\sigma^2)$
  2. $\sigma^2 | Y, \beta, X \sim IG(\frac{T+k}{2} + v_0, \frac{\sum_{t=1}^{T} \epsilon_t^2}{2} + s_0)$

where $B_0 = [0 \ \ \beta_0 \ \ 0 \ \ 0]$ and $M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Posterior Distribution

$$f(\beta, \sigma^2 | Y, X) \propto f(Y|\beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Conjugate prior : $\beta_1|\sigma^2 \sim N(\beta_0, m_0^{-1}\sigma^2)$ and $\sigma^2 \sim IG(v_0, s_0)$

- Gibbs sampling
  1. $\beta | Y, \sigma^2, X \sim N((X'X + M_0)^{-1}(X'y + M_0 B_0), (X'X + M_0)^{-1}\sigma^2)$
  2. $\sigma^2 | Y, \beta, X \sim IG(\frac{T+k}{2} + v_0, \frac{\sum_{t=1}^{T} \epsilon_t^2}{2} + s_0)$

  where $B_0 = [0 \ \ \beta_0 \ \ 0 \ \ 0]$ and $M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & m_0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

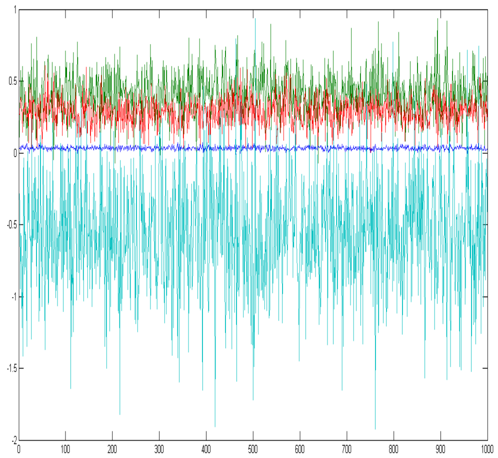
Set $\beta_0 = 0$, $m_0 = (0.1^2)^{-1}(\frac{s_0}{v_0 - 2})$, $s_0 = \frac{(Y - X\beta_{\text{ols}})'(Y - X\beta_{\text{ols}})}{2}$, $v_0 = 3/2$

Posterior Distribution

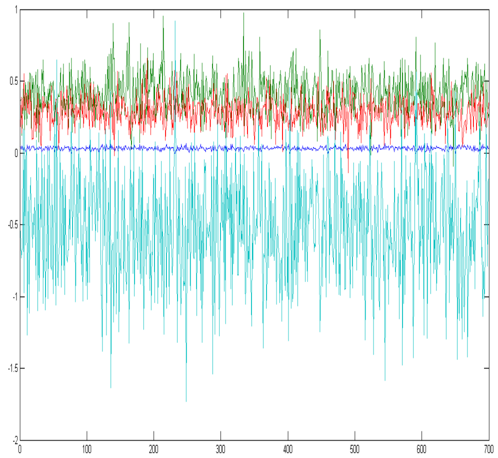
$$f(\beta, \sigma^2 | Y, X) \propto f(Y | \beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Results :

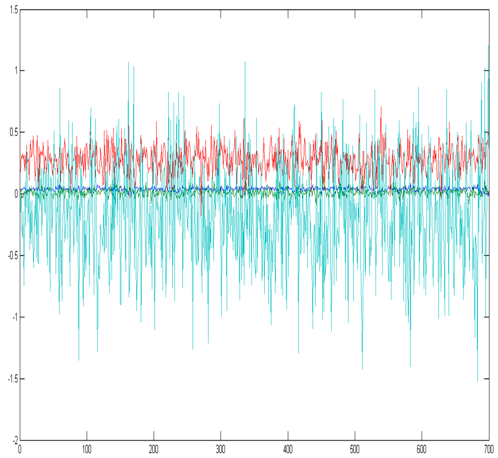
```
[Simu] = Gibbs_regression(y,X,10000,2,1)
plot(Simu.DS_beta')
plot(Simu.post_beta')
```



DS (diffuse)



Gibbs (diffuse)



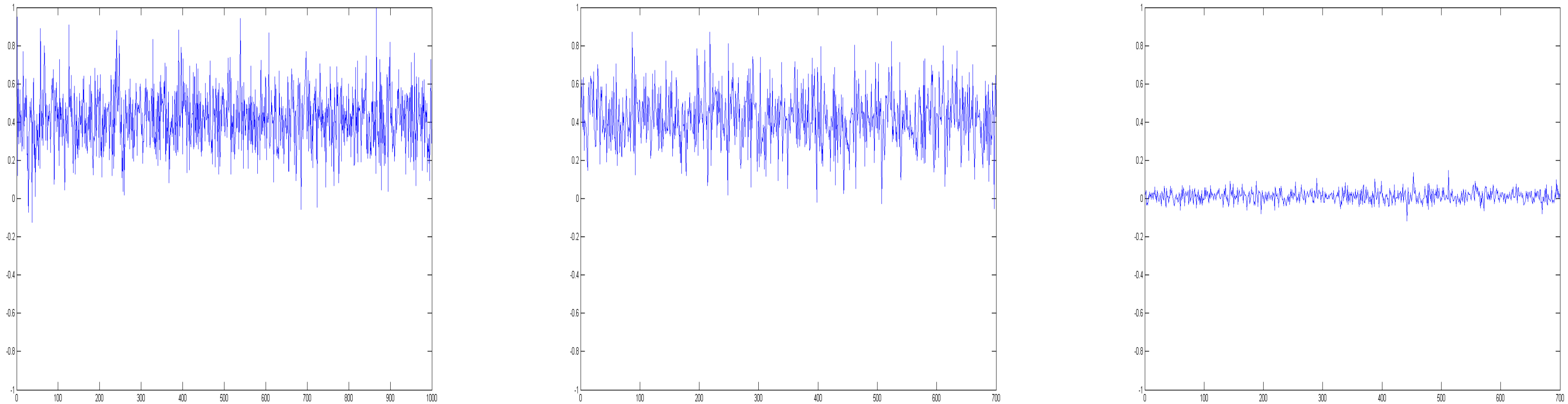
Gibbs (Conjugate)

Posterior Distribution

$$f(\beta, \sigma^2 | Y, X) \propto f(Y | \beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Focus on β_1

```
plot(Simu.DS_beta(2,:))
plot(Simu.post_beta(2,:))
```



DS (diffuse) Gibbs (diffuse) Gibbs (Conjugate)

Posterior Distribution

$$f(\beta,\sigma^2|Y,X) \propto f(Y|\beta,\sigma^2,X)f(\beta,\sigma^2|X)$$

Normal prior for β_1 and diffuse for the other parameters :

$$f(\beta,\sigma^2) \propto f(\beta_1)\frac{1}{\sigma^2}$$

where $\beta_1 \sim N(\beta_0, M_0^{-1})$

- Not Conjugate but Gibbs sampling
  1. $\beta|Y,\sigma^2,X \sim N((X'X+M_0\sigma^2)^{-1}(X'y+M_0B_0),(X'X+M_0\sigma^2)^{-1}\sigma^2)$
  2. $\sigma^2|Y,\beta,X \sim IG(\frac{T}{2},\frac{\sum_{t=1}^{T}\epsilon_t^2}{2})$

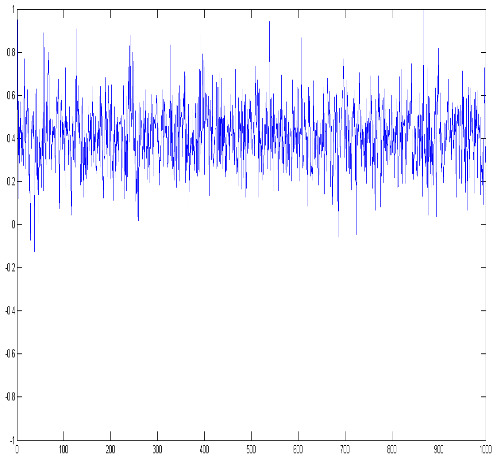
  where $B_0 = [0 \;\; 0 \;\; 0 \;\; 0]$ and $M_0 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.1^{-2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

Posterior Distribution

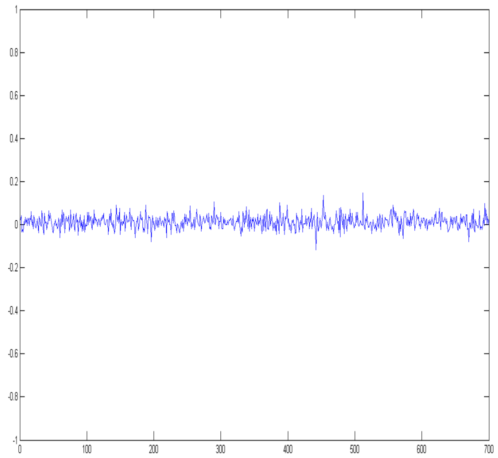
$$f(\beta, \sigma^2 | Y, X) \propto f(Y | \beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Focus on β_1

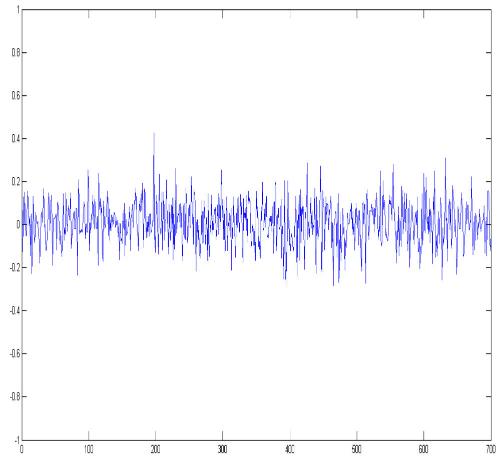
```
[Simu] = Gibbs_regression(y,X,10000,3,1)
plot(Simu.DS_beta(2,:))
plot(Simu.post_beta(2,:))
```



DS (diffuse)



Gibbs (Conjugate)



Gibbs (Not-Conjugate)

Posterior Distribution

$$f(\beta, \sigma^2 | Y, X) \propto f(Y | \beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Beta prior for β_1 and diffuse for the others :

$$f(\beta, \sigma^2) \propto f(\beta_1) \frac{1}{\sigma^2}$$

where $\beta_1 \sim$ symmetric Beta distribution on $[-0.3, 0.3]$

$$f(\beta | a) = \frac{\Gamma(2a)}{\Gamma(a)^2} (2c)^{2a-1} [(y+c)(y-c)]^{a-1}$$

where $c = 0.3$ and $a = 0.59$

- Not conjugate and no Gibbs sampler
  1. $\beta | Y, \sigma^2, X \sim ?$ Sampling By MH
  2. $\sigma^2 | Y, \beta, X \sim IG(\frac{T}{2}, \frac{\sum_{t=1}^{T} \epsilon_t^2}{2})$

Posterior Distribution

$$f(\beta,\sigma^2|Y,X) \propto f(Y|\beta,\sigma^2,X)f(\beta,\sigma^2|X)$$

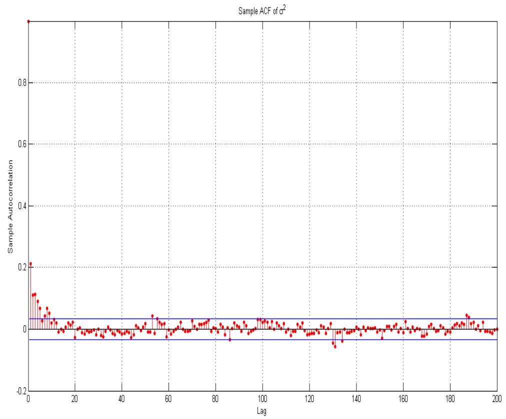
[Simu] = MH_regression(y,X,nb_MCMC,graph)

1. y and X : dependent and explanatory variables.
2. nb_MCMC : number of MCMC iterations.
3. graph : = 1 displays some convergence graphics.

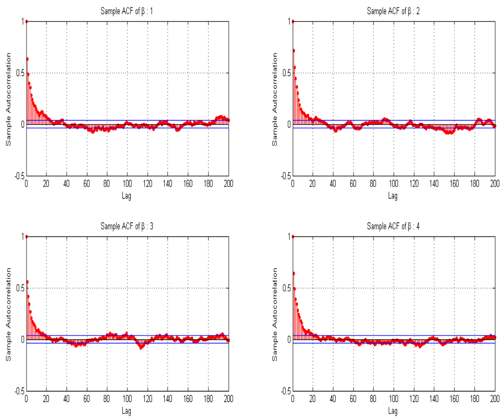
To run the function :

[Simu] = MH_regression(y,X,5000,1)

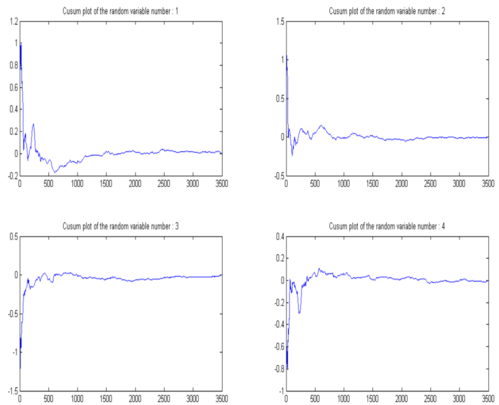
[Simu] = MH_regression(y,X,5000,1)



ACF σ^2



ACF β



Cusum plot β

A structure Simu

```
*********************************
*****  BEGINNING OF THE MCMC  ******
*********************************

Simu =

      post_beta:  [4x3500 double]
     post_sigma:  [3500x1 double]
       beta_ols:  [4x1 double]
      sigma_ols:  0.00
         accept:  0.39
      prior_a_c:  [0.59 11.00]
    test_Geweke:  [4x1 double]
          cusum:  [1x1 struct]

>> |
```

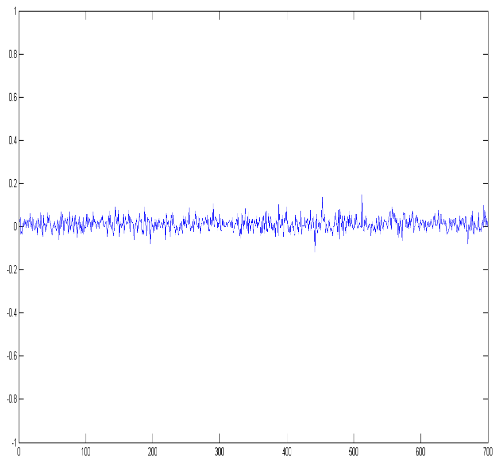
Structure

Posterior Distribution

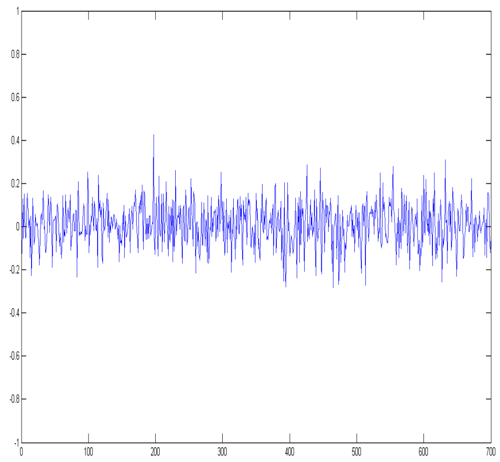
$$f(\beta, \sigma^2 | Y, X) \propto f(Y|\beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Focus on β_1

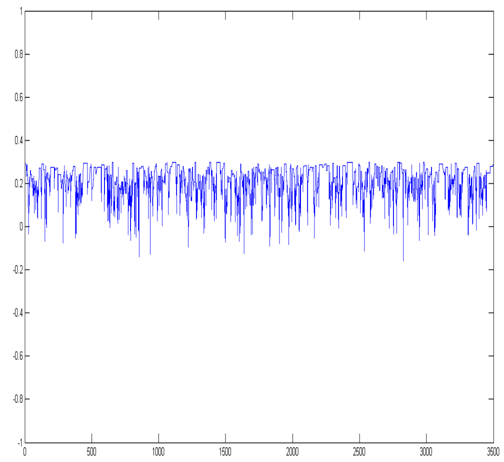
```
plot(Simu.post_beta(2,:))
```



Gibbs (Conjugate)



Gibbs (Not-Conjugate)



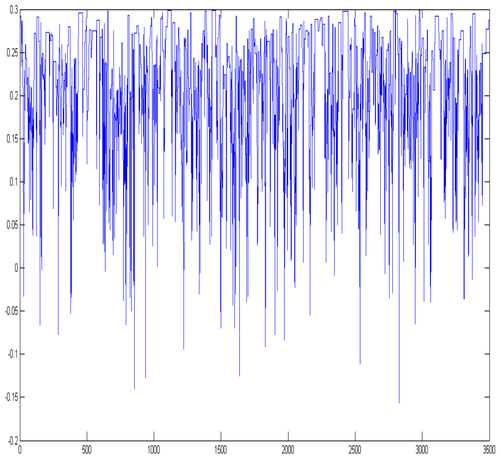
MH (beta)

Posterior Distribution

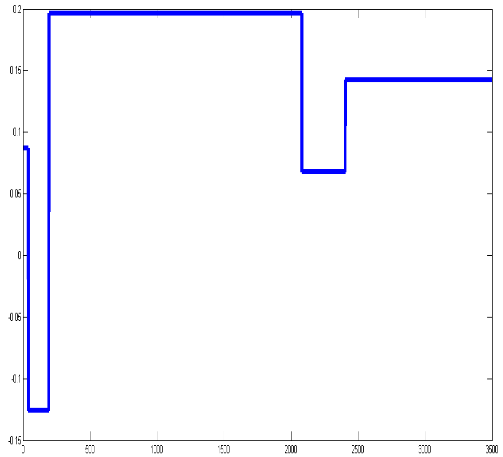
$$f(\beta, \sigma^2 | Y, X) \propto f(Y | \beta, \sigma^2, X) f(\beta, \sigma^2 | X)$$

Focus on β_1

```
Sigma_beta = 100*Sigma_cond*sigma;
plot(Simu.post_beta(2,:))
```



MH (Beta)



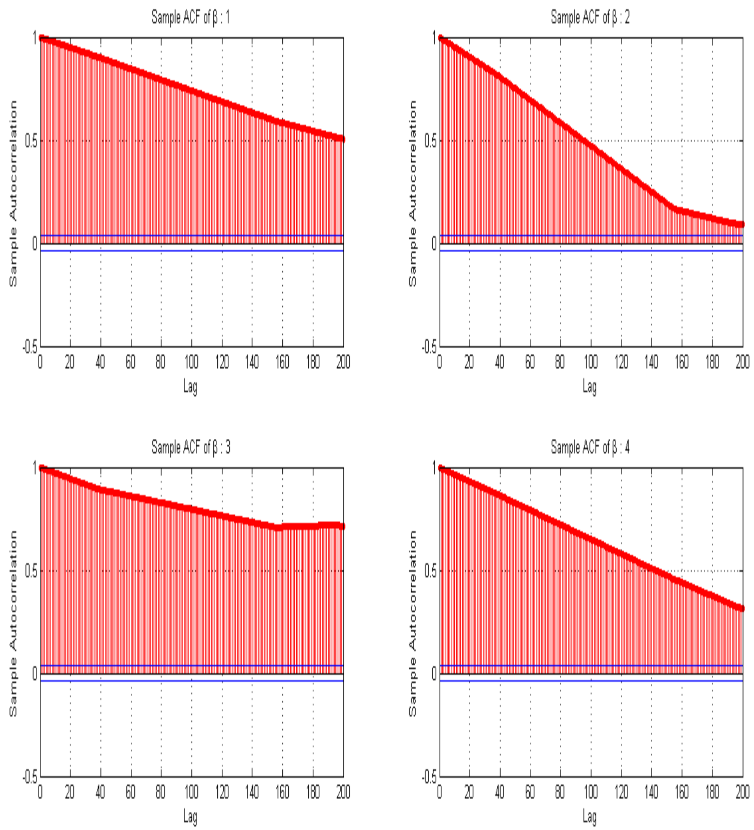
MH (Beta) (Higher variance)

acc. rate = 0.39

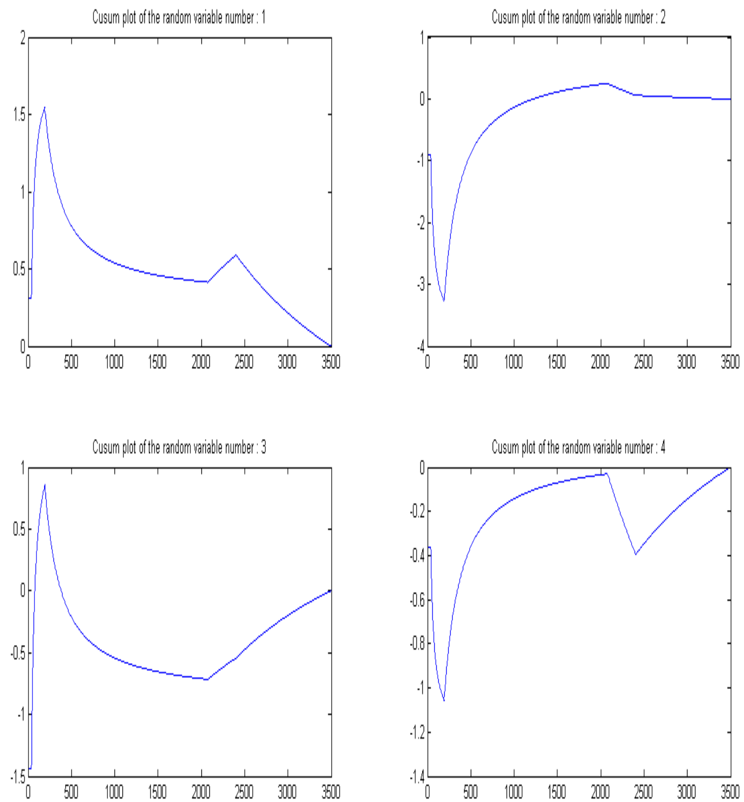
acc. rate = 0.001

[Simu] = MH_regression(y,X,5000,1)

```
Sigma_beta = 100*Sigma_cond*sigma;
```



ACF β



Cusum plot β