Matlab Tutorials

April 2013

Tutorial 3 on Matlab

Solutions are provided in the directory Tutorial3Solution :

- Question 1 : MCMC_VAR.m

Running a Matlab function :

- Click on the m file that you want to run
- Copy and paste in the 'command window' the first line without the word 'function'
- Choose adequate inputs for the function.
- For example (see MCMC_VAR.m) :

[Simu] = MCMC_VAR(y,6,5000,0,1)

Tutorial 3 on Matlab

Bayesian inference for unrestricted VAR model

$$r_t = c_1 + \sum_{i=1}^{6} \beta_{1,i} r_{t-i} + \sum_{i=7}^{12} \beta_{1,i} R_{t-i+6} + \epsilon_{1,t} \quad (1)$$

$$R_t = c_2 + \sum_{i=1}^{6} \beta_{2,i} r_{t-i} + \sum_{i=7}^{12} \beta_{2,i} R_{t-i+6} + \epsilon_{2,t} \quad (2)$$

with $\epsilon_t = [\epsilon_{1,t} \;\; \epsilon_{2,t}] \sim N(0, \Sigma)$

The data are available in the mat file data_tuto3.mat

- Monthly, covering the period 1960.1 until 1996.12 (444 observations).
- The mat file contains

$$y = [r_t \;\; R_t]$$

- Parameter set : $\forall i \in [1, 2] \;\; and \;\; j \in [1, 11] \;\; \beta = (c_1 \;\; c_2 \;\; \beta_{j,i})'$ and Σ

Posterior Distribution

$$f(\beta, \Sigma | Y) \propto f(Y|\beta, \Sigma) f(\beta, \Sigma)$$

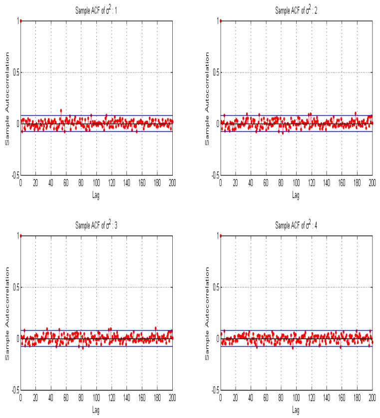
[Simu] = MCMC_VAR(y,lags,nb_MCMC,minnesota,graph)

1. y : dependent and explanatory variables.
2. lags : number of lags in the VAR regression
3. nb_MCMC : number of MCMC iterations.
4. minnesota : if =0 then NIP prior otherwise =1 : Minnesota prior
5. graph : = 1 displays some convergence graphics.

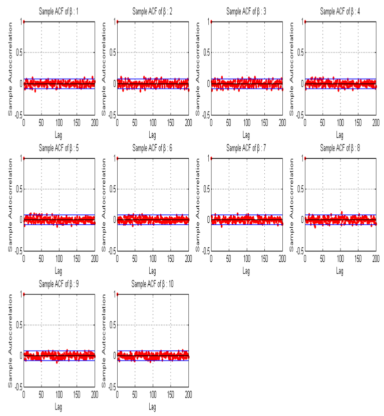
To run the function :

[Simu] = MCMC_VAR(y,2,1000,0,1)

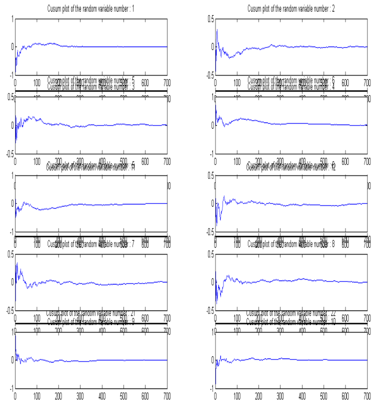
[Simu] = MCMC_VAR(y,2,1000,0,1)



ACF Σ



ACF β



Cusum plot β

A structure Simu

```
Simu =

                DS_beta: [10x1000 double]
               DS_Sigma: [4x1000 double]
              post_beta: [10x700 double]
             post_Sigma: [4x700 double]
               sum_beta: [4x700 double]
            eigen_value: [700x1 double]
        post_mean_eigen: 0.00
         post_std_eigen: 0.00
         post_mean_beta: [10x1 double]
          post_std_beta: [10x1 double]
     post_mean_sum_beta: [4x1 double]
      post_std_sum_beta: [4x1 double]
          post_mean_std: [4x1 double]
           post_std_std: [4x1 double]
               post_rho: [1x700 double]
          post_mean_rho: 0.34
           post_std_rho: 0.03
            test_Geweke: [10x1 double]
                  cusum: [1x1 struct]
```

Structure

A structure Simu

```
Simu =

              DS_beta: [10x1000 double]
             DS_Sigma: [4x1000 double]
            post_beta: [10x700 double]
           post_Sigma: [4x700 double]
             sum_beta: [4x700 double]
          eigen_value: [700x1 double]
      post_mean_eigen: 0.00
       post_std_eigen: 0.00
       post_mean_beta: [10x1 double]
        post_std_beta: [10x1 double]
   post_mean_sum_beta: [4x1 double]
    post_std_sum_beta: [4x1 double]
        post_mean_std: [4x1 double]
         post_std_std: [4x1 double]
             post_rho: [1x700 double]
        post_mean_rho: 0.34
         post_std_rho: 0.03
         test_Geweke: [10x1 double]
                cusum: [1x1 struct]
```

To get one field of the structure : In the command window

Simu.post_mean_beta
Simu.post_std_beta
Simu.post_mean_rho
Simu.post_std_rho

Posterior Distribution

$$f(\beta, \Sigma | Y) \propto f(Y | \beta, \Sigma) f(\beta, \Sigma)$$

Two different priors :

1. Diffuse prior (Direct Sampling and Gibbs sampling is possible)

$$f(\beta, \Sigma) \propto |\Sigma|^{-\frac{n+1}{2}}$$

2. Minnesota prior (Only Gibbs sampling)

$$f(\beta, \Sigma) \propto f(\beta) f(\Sigma)$$

where $\beta | \Sigma \sim$ Minnesota prior and $\Sigma \propto |\Sigma|^{-\frac{n+1}{2}}$

Posterior Distribution

$$f(\beta, \Sigma | Y) \propto f(Y|\beta, \Sigma) f(\beta, \Sigma)$$

The model can be re-written as

$$Y = ZB + E$$

Diffuse prior (Direct Sampling and Gibbs sampling is possible) :

$$f(B, \Sigma) \propto |\Sigma|^{-\frac{n+1}{2}}$$

1. Direct Sampling :

$$\begin{aligned} f(B, \Sigma | Y) &= f(B|Y, \Sigma) f(\Sigma | Y) \\ &= MN(\hat{B}, \Sigma \otimes (Z'Z)^{-1}) \;\; IW(T-k, S) \end{aligned}$$

where

$$\begin{aligned} \hat{B} &= (Z'Z)^{-1} Z'Y \\ S &= Y'Y - Y'Z(Z'Z)^{-1} Z'Y \end{aligned}$$ (see slide 156)

Posterior Distribution

$$f(B,\Sigma|Y) \propto f(Y|B,\Sigma)f(B,\Sigma)$$

1. Direct Sampling :

$$\begin{aligned} f(B,\Sigma|Y) &= f(B|Y,\Sigma)f(\Sigma|Y) \\ &= MN(\hat{B},\Sigma\otimes(Z'Z)^{-1})\;\; IW(T-k,S) \end{aligned}$$

where

$$\begin{aligned} \hat{B} &= (Z'Z)^{-1}Z'Y \\ S &= Y'Y - Y'Z(Z'Z)^{-1}Z'Y \quad \text{(see slide 156)} \end{aligned}$$

```
for i=1:nb_MCMC
   Sigma = iwishrnd(S,df_IW);
   Sigma_star = tidy_cov_mat(kron(Sigma,inv_X_vec));
   Simu.DS_beta(:,i) = mvnrnd(beta_ols_vec,Sigma_star)';
   Simu.DS_Sigma(:,i) = reshape(Sigma,dimension^2,1);
end
```

Posterior Distribution

$$f(\beta, \Sigma | Y) \propto f(Y | \beta, \Sigma) f(\beta, \Sigma)$$

Under Diffuse prior

1. Gibbs sampler :

$$f(B | Y, \Sigma) \sim MN(\hat{B}, \Sigma \otimes (Z'Z)^{-1})$$
$$f(\Sigma | Y, B) \sim IW(T, (Y - ZB)'(Y - ZB))$$

where

$$\hat{B} = (Z'Z)^{-1} Z'Y$$

Sampling of $\Sigma | Y, B$:

```
eps = Sigma_bar;
for t=1:T
    aide = X_stack(:,:,t)*beta;
    eps = eps +  (y(t,:)'-aide)*(y(t,:)-aide');
end
inv_post = inv(eps);
Sigma_inv = wishrnd(inv_post,T+df_Sigma);
Sigma = inv(Sigma_inv);
```

Posterior Distribution

$$f(B,\Sigma|Y) \propto f(Y|B,\Sigma)f(B,\Sigma)$$

Gibbs sampler Under Diffuse prior

Sampling of $B|Y,\Sigma$:

```
Sigma_star = tidy_cov_mat(kron(Sigma,inv_X_vec));
beta = mvnrnd(beta_ols_vec,Sigma_star)';
```

Initial point of the MCMC

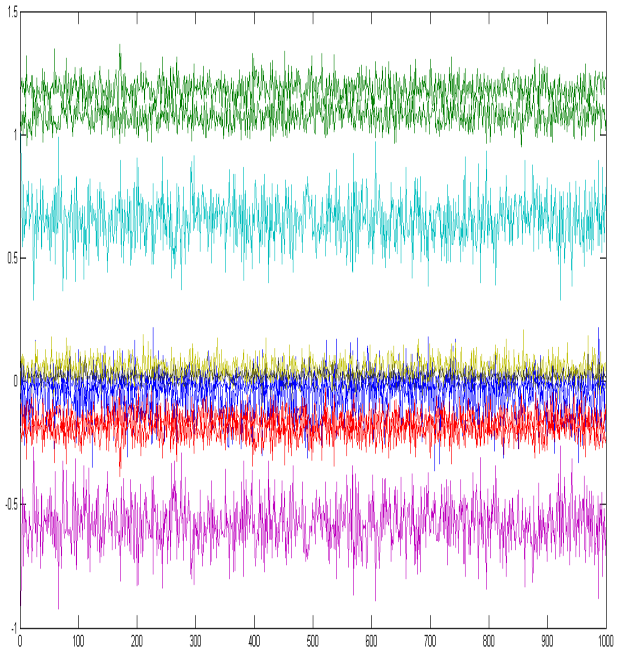
- Parameters that exhibit the highest possible posterior density
- For example : Ordinary least square or MLE

Posterior Distribution

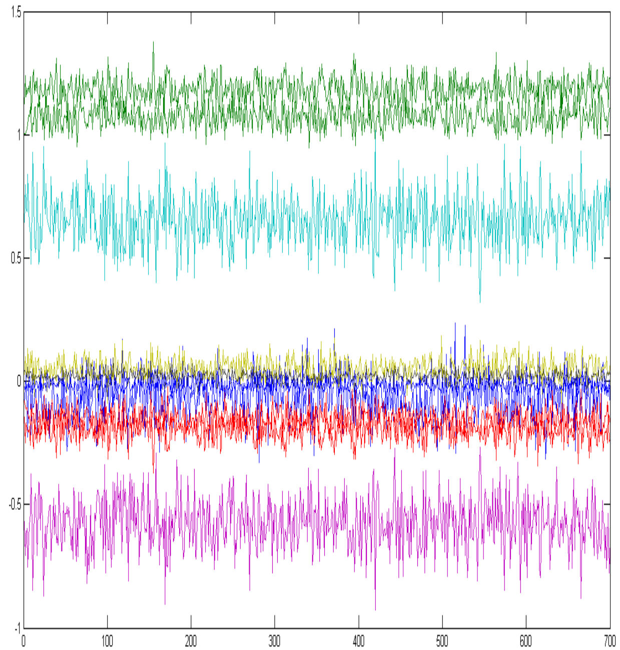
$$f(B, \Sigma | Y) \propto f(Y | B, \Sigma) f(B, \Sigma)$$

Results :

```
[Simu] = MCMC_VAR(y,2,1000,0,0)
plot(Simu.DS_beta')
plot(Simu.post_beta')
```



B - DS (diffuse)



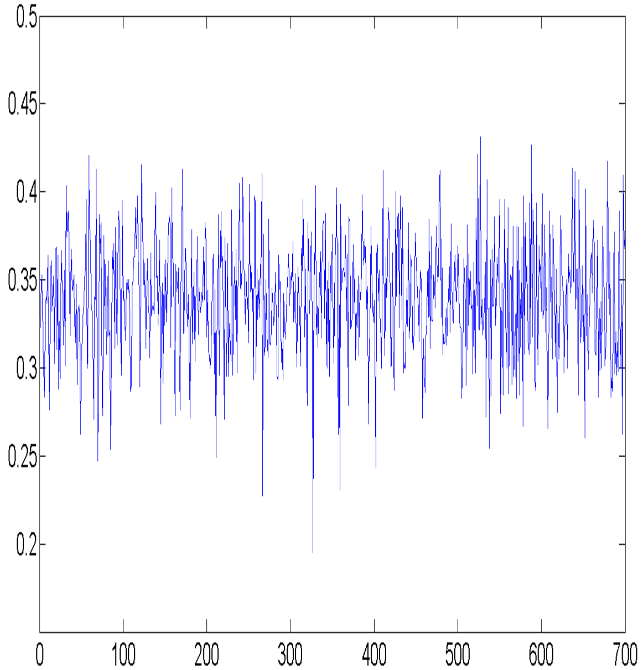
B - Gibbs (diffuse)

Posterior Distribution

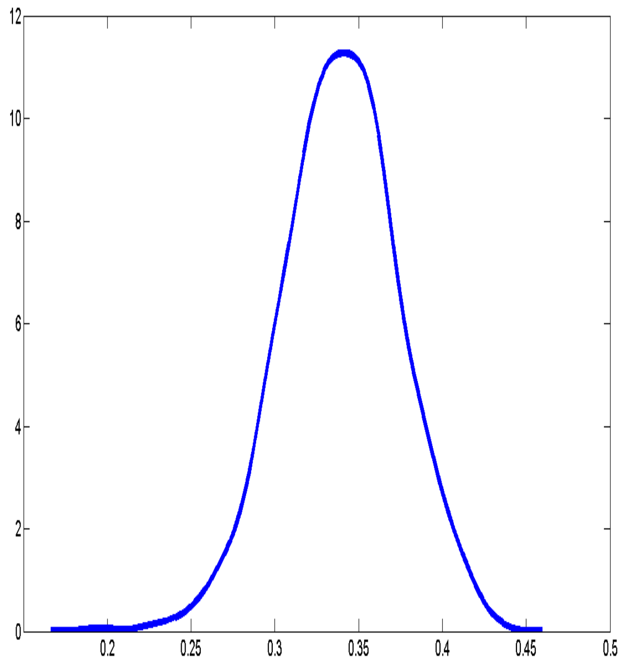
$$f(B,\Sigma|Y) \propto f(Y|B,\Sigma)f(B,\Sigma)$$

Results :

```
[Simu] = MCMC_VAR(y,2,1000,0,0)
plot(Simu.post_rho')
```



rho - Gibbs (diffuse)



rho - Gibbs (diffuse)

Posterior Distribution

$$f(B,\Sigma|Y) \propto f(Y|B,\Sigma)f(B,\Sigma)$$

Example on the first equation - Minessota prior :

$$r_t = c_1 + \sum_{i=1}^{6} \beta_{1,i} r_{t-i} + \sum_{i=7}^{12} \beta_{1,i} R_{t-i+6} + \epsilon_{1,t} \tag{3}$$

- First lag of the dependent variable : $\beta_{1,1} \sim N(1,\lambda)$
- Other lags of the dependent variable : $\beta_{1,i} \sim N(0,\frac{\lambda}{i})$ with $i < 7$
- Lags of the other variable : $\beta_{1,i} \sim N(0,\theta\frac{\lambda}{i-5}\frac{\sigma_{r_t}}{\sigma_{R_t}})$ with $i > 6$

Minnesota prior : vec $B \sim N(\text{vec } B_0, M_0^{-1})$

Posterior Distribution

$$f(B, \Sigma | Y) \propto f(Y | B, \Sigma) f(B, \Sigma)$$

Minessota prior : Gibbs Sampler

$$\Sigma | Y, B \sim IW(T - k + n, (Y - ZB)'(Y - ZB))$$
$$\text{Vec } \ B | Y, \Sigma \sim N(\text{vec } \ B_*, M_*^{-1})$$

where (see slide 169)

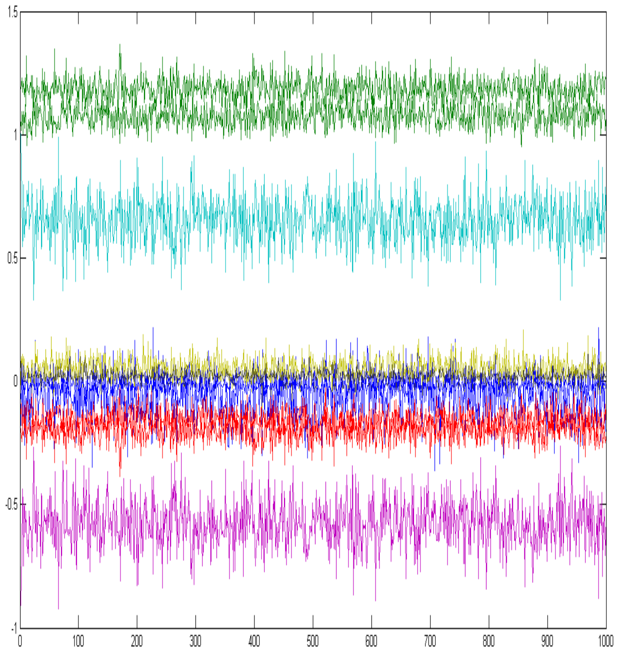
$$M_* = \Sigma^{-1} \otimes Z'Z + M_0$$
$$\text{Vec } \ B = M_*^{-1}[(\Sigma^{-1} \otimes Z'Z)\text{vec } \ \hat{B} + M_0 \text{vec } \ B_0]$$

Posterior Distribution

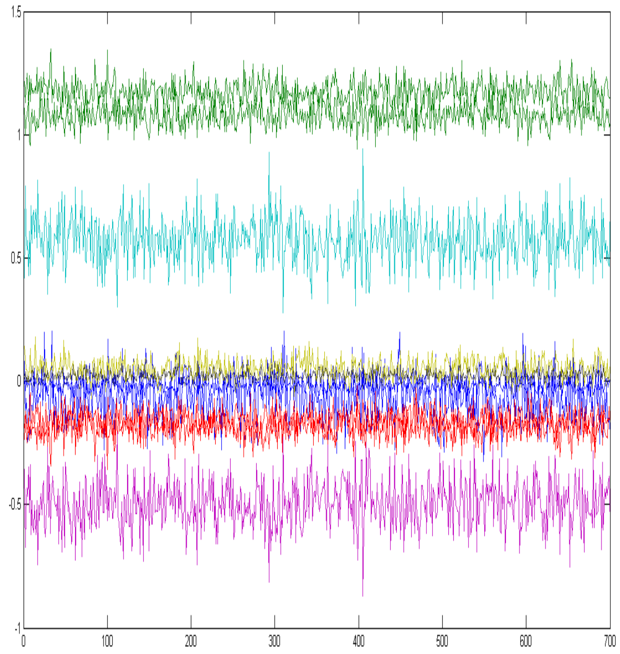
$$f(B,\Sigma|Y) \propto f(Y|B,\Sigma)f(B,\Sigma)$$

Results :

```
[Simu] = MCMC_VAR(y,2,1000,1,0)
plot(Simu.DS_beta')
plot(Simu.post_beta')
```



B - DS (diffuse)



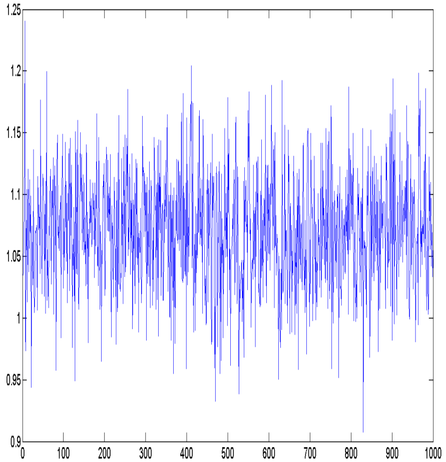
B - Gibbs (Minnesota)

Posterior Distribution

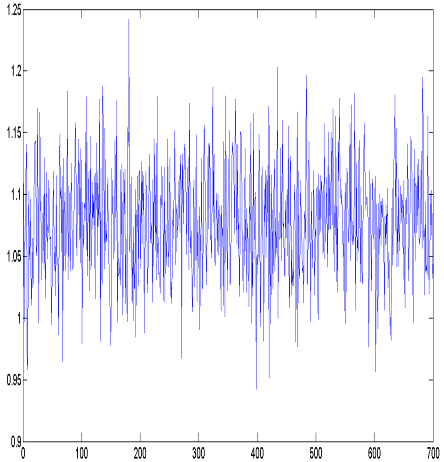
$$f(B, \Sigma | Y) \propto f(Y | B, \Sigma) f(B, \Sigma)$$

Results :

```
[Simu] = MCMC_VAR(y,2,1000,1,0)
plot(Simu.DS_beta(2,:))
plot(Simu.post_beta(2,:))
```



$\beta_{1,1}$ - DS (diffuse)



$\beta_{1,1}$ - Gibbs (Minnesota)

Posterior means of $\beta_{1,1}$ under the two priors

```
mean(Simu.DS_beta(2,:))
mean(Simu.post_beta(2,:))
```

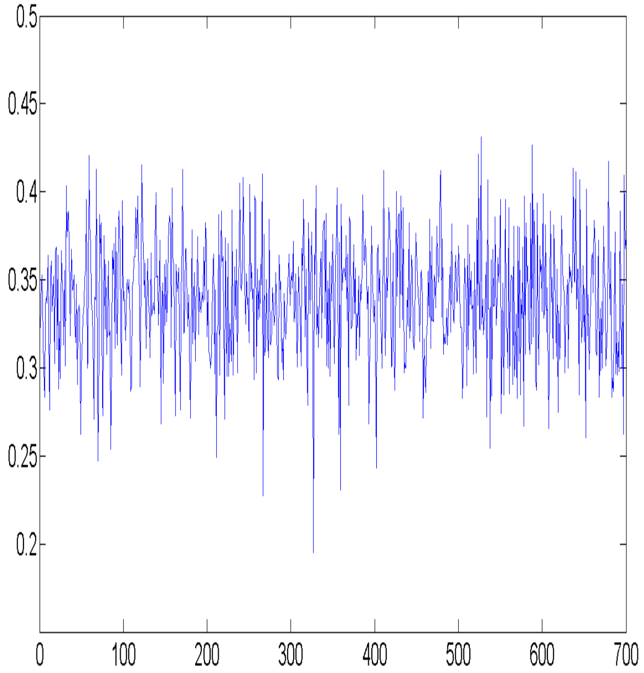
1.07 1.08

Posterior Distribution

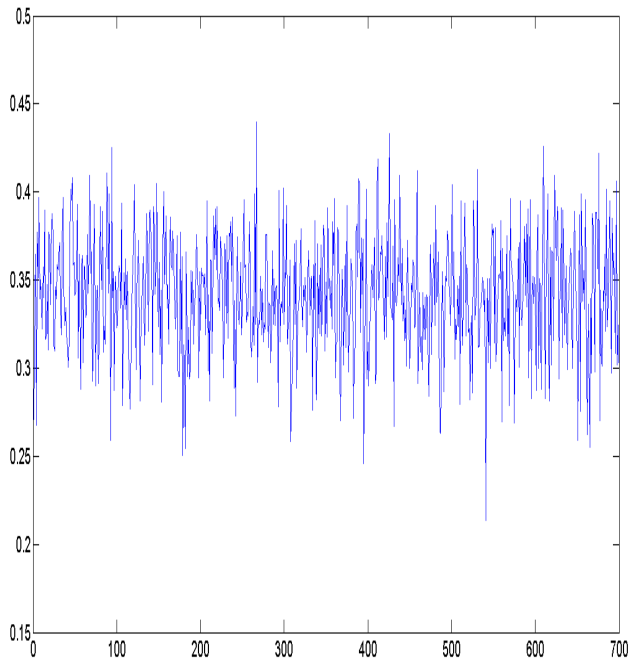
$$f(B, \Sigma | Y) \propto f(Y | B, \Sigma) f(B, \Sigma)$$

Results :

```
[Simu] = MCMC_VAR(y,2,1000,1,0)
plot(Simu.post_rho')
```



rho - Gibbs (diffuse)



rho - Gibbs (Minnesota)