The role of wage formation in empirical macroeconometric models

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Abstract

The paper surveys the main lines for treatment of wage formation followed by macro econometric model builders since the 1950s. The main models are the Phillips Curve (PCM), the wage-price equilibrium correction model (WP-ECM) and the New Keynesian Phillips curve (NKPCM). The models can be represented as different specifications of a system of difference equations that define the supply side of a medium term macro econometric model. The econometric treatment of wage formation therefore affects the dynamic solutions of supply side variables and in principle all the endogenous variables of the model. The model dependency of the concept of an equilibrium rate of unemployment is used as an main example in the article. Moreover, other properties of the models are closely connected to their solution: Dynamic multipliers (and impulse responses) get their main features from the homogenous part of the solution. The same is the case for model based dynamic forecasts. The specifications of wage and price formation therefore affect both model based policy analysis and macroeconomic projections. Examples are given of models of wage formation in economies with different systems of wage formation, e.g., with respect to the importance of collective bargaining in wage regulation.

1 Introduction

The properties of a macro model reflect the specification of both the demand and the supply side of the economy. As explained by Nickell (1988) the key part of the supply-side are represented

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by those equations that describe the behaviour of firms, in particular price setting, and those that reflect the determination of wages.

In the following, the term wage formation will be used relatively broadly, referring to the process whereby the renumeration of wage earners is determined. Negotiations about wage adjustments and working conditions can take place between individuals and firms, or it can be of a collective nature, which in turn can be at level of the firm, industry level or even at the national level. In a given historical epoch, national systems of wage formation is mixture of individual negotiations, collective bargaining and regulation by laws. However, the different principles of wage adjustment are usually not equally important. Hence, when we refer to systems of collective wage formation in the following, we have in mind systems of national wage formation where collective agreements are negotiated in a majority of industries and sectors and regulate payment and working conditions for a significant part of the employed wage earners. In many countries with collective wage bargaining there are also extension mechanisms, implying that collective agreement are extended to (maybe) large numbers of wage earners who do not hold a union membership. This means that the coverage of collective agreements about wage compensation and working time can exceed the degree of unionization in some countries. The role of extension mechanism may have increased in recent years, and it raises several questions about the future role of collective labour contracts, which are however beyond the scope of this article.

Wage formation is an integral part of processes that are central to the performance of economies. Wage income is the largest component of households' disposable income, which affects aggregated spending and private savings. The wage distribution is important for overall inequality in the economy. Capital income is typically more unequally distributed than wage income. This means that a change in the functional income distribution, from wages to capital, has additional effects on inequalities of modern societies. Neither are mechanisms behind the two distributions completely separate. It is plausible that the system of wage formation and labour market institutions can influence the functional income distribution.

The wage per unit of labour used in the production of goods and services is an important cost components of firms. Since firms also use labour indirectly, through the purchase of intermediate goods, the share of wages in production costs tends to increase with the level of aggregation. Firms deal with increased wage costs in several ways, including mark-up of prices, adaption of labour saving technologies, out-sourcing and other strategies. Sometimes, this has consequences at the macro level. For example a weakening of the trade balance and an increase in the level of unemployment. Yet, as any student of economics will know, a general wage reduction is not necessarily the right solution to an unemployment crisis, because the wage level is also important for activity level through aggregate demand. The effects of wage changes are multiple, and they may go in opposite directions.

Since the general wage level enters into some of the most central processes of modern economies, it follows that the representation of wage formation becomes important in macroeconomic models that are produced to represent real world economies in simplified form, so that they can be analyzed and forecasted.

In the medium term time perspective, wage formation plays an important role in how shocks to the economy are propagated in the economy. As we explain below, the decision about how to represent wage formation has implications for the modeling of wage changes over the business cycle, as procyclical or countercyclical.

The propagation of negative shocks to the macro economy needs not imply a relatively short business cycle, the results might be a stagnation period (long-cycle), and the system of wage formation is seen as important for that possibility as well.

In this article one focus will be the role of wage formation in the determination of a neutral rate of unemployment level. By a "neutral rate" we mean a steady-state level of unemployment implied by the macro model under consideration. The term refers to a broader concept than the "natural rate" of unemployment, which when made precise is seen to depend on a sub-set of supply side factors. The wider conceptualizaton is required for models which have the property that wage inflation can be constant (in the steady-state sense) conditional on a *set* of unemployment rates. Specifically, more factors need to be brought into the picture to determine a unique neutral rate when wage formation is modelled as a system of collective bargaining, than is needed to pin down the natural rate implied by individual negotiations modelled by Phillips curves.

Closely connected to this are policy issues. Like: How active the government can be in the provision of monetary and fiscal stimulus without fuelling inflation or contributing to trade balance problems through wage-price and wage-wage spirals. Models of price setting and of wage formation have relevance for these issues, because they imply strategies for feasible good economic performance. The idea about a natural rate of unemployment (sometimes just referred to as "u-star" u^*)

has been particularly successful in influencing monetary policy, but may have led to consistent underestimation of slack in e.g., USA's labour markets, Solow (1999). However, the fortunes of ideas and models change over time. Recently, Federal Reserve Chair Jerome Powell spoke about a new medium term monetary policy strategy, going for maximum employment as opposed to offsetting deviations from assessments of u^*, Powell (2020).

As indicated in the title, the aim of this article is to discuss wage formation in empirical models of the macro economy, as opposed to theoretical models. Hence we have in mind models of real world economies, with estimated coefficients, and with statistical properties (needed for inference) that depend on how well the specifications of the model equations individually and jointly capture an unknown data generating process (DGP).

In the search of not-misspecified empirical macroeconometric models, econometricians make use of both statistical models and economic theories, often in an eclectic way since neither of these model classes are complete or general, or immediately relevant for real world data. In the simplest case, where the statistical model is a regression model, the consequence of the position that a unknown DGP has generated the observations is that the disturbance is a derived variable, namely:

$$\underset{\text{observed}}{Y_t} = \underset{\text{explained}}{f(X_t)} + \underset{\text{remainder}}{\varepsilon_t} \tag{1}$$

where Y_t are the observations of the dependent variable which we seek to explain by the use of economic theory and our knowledge of the subject matter.

Our explanation is given by the function $f(X_t)$, In the regression case, it is the conditional expectation function, but also in that simple interpretation the explanation is the result of a range of decisions, including variable selection and functional form, Hendry (2018). The non-experimental Y_t is not determined or caused by $f(X_i)$, it is determined by a DGP that is unknown for us, and all variation in Y_i that we do not account for, must therefore "end up" in the remainder ε_t.

Hence, the situation is different from the one the experimental researcher is in, which can be represented as:

$$\underset{\text{result}}{Y_t} = \underset{\text{input}}{g(X_t)} + \underset{\text{shock}}{v_t} , \tag{2}$$

see Hendry (1995a, Ch 1.11). The variable Y_t is this case interpretable as the result of the experiment, while the X_t is the imputed input variable which is decided by the researcher. $g(X_t)$ is a deterministic function. The variable v_t is a shock which leads to some separate variation in Y_i for the chosen X_i.

Again, unlike (2), where v_t represents free and independent variation to Y_t, ε_t in (1) is an *implied* variable which gets its properties from the DGP and the explanation, in effect from the substantive model (explanation) $f(X_t)$. Hence, for an empirical econometric model, we should write:

$$\varepsilon_t = Y_t - f(X_t) \tag{3}$$

to describe that whatever we do on the right hand side of (3) by way of changing the specification of $f(X_t)$ or by changing the measurement of Y_t, the left-hand side is derived as a result, Hendry (1995a, Ch. 2.27).

In the non-experimental situation, the specification of the statistical model of ϵ_t is conditional on X_t and the functional form. Since the DGP is unknown, the assumed statistical model needs to be tested for mis-specification, see e.g. Spanos (2020), Nymoen (2019, Ch. 2.8) among others. The feasibility of mis-specification testing rests on the properties of the residuals $\hat{\epsilon_t}$ as valid statistics of the remainder in (1). If any of the assumptions of the statistical model are found to be indefensible after testing, one needs to re-specify the explanation (change the explanatory variables and/or functional form) until statistical adequacy can be demonstrated. At least ideally. In practice one may have to stop the process of re-specification before statistical adequacy is complete, and in that case one has to keep in mind the consequences that the remaining mis-specification has for the estimation of (the parameters of) $f(X_t)$.

A macroeconomic model which has been completely specified from theory is oddly in analogy with (2), but only by invoking the *axiom of correct specification* (Leamer (1983)), rather than with the empirical model (1). The fact that such a model may use and produce numerical data, we can refer to it as a quantitative model, does not change that conclusion, since the result is a consequence of the asserted input and the assumed probability distribution of the shocks. Unless the assertions and assumptions that underlie the quantitative model are confronted with the data, and revised in the light of any evidence that make them indefensible, can we speak of a genuine empirical model.

That said, producers of empirical macro models have always made use of economic theory in the model specification process. This is both to give economic interpretability and to clarify the dynamic properties of the model (or in practice, a module of the complete model), see e.g., Bårdsen et al. (2005, Ch. 3). Hence, the difference is not so much whether theory is seen as useful or not for the model specification. The difference is more the attitude towards data confrontation and mis-specification testing. At the end of the day, an empirical macro model will always be a result of several compromises.

2 Lineages of wage modelling in macro economics

The representation of wage formation in macroeconometric models has changed over time. During the first epoch of macroeconomic modelling, theoretical and empirical modellers took a cautious approach to wage formation. The going economic theory of the wage variable in the 1950s has been characterized as a certain exogeneity proposition about the wage level, Forder (2014, Ch. 1). It was however not the recognition that the wage level was seen as unconditionally fixed. Rather, it was a proposition about a exogeneity of nominal wages and prices to the level of unemployment.

Hence, the theoretical view was twofold: First that unemployment could vary quite a lot without any very noticeable effects on wages and prices in macro. This was later known as the 'L-shaped' wage (and price) curve. Second, that wages could increase (or be reduced) a good deal without any simultaneous or preceding change in unemployment taking place. In turn, this conceptualization opened for a clear understanding of cost-push elements and possibility of wage and price spirals.

The exogeneity proposition that was characteristic of applied macro in the 1950s and 1960s was therefore not a sign of lack of thinking about wage formation. On the contrary, it was an expression of how far economics had come in finding a relevant theoretical perspective on wage setting.

Neither was this a new recognition. Ragnar Frisch, who's contributions during the 1930s had defined the field of macroeconomic dynamics, lamented in 1945 about the lack of a scientific representation of the role of wage formation in the macro economic system:

> A full analysis of how the wage level enters into this complex system of causes, with a numerical representation of the strength of the relationships, can unfortunately not be presented. This is one of the most unfortunate holes in the economic science. Ragnar Frisch (Arbeiderbladet 30 August 1945).

Frisch' sense of a certain failure on behalf of the economics discipline was probably shared my many. But other leading economists could better "live with" the situation. It was just a realistic view that used to be widely accepted, as in Samuelson's textbook (3ed 1955, p 547):

> [wage formation]...depends on psychology, politics, and thousands of other intangible factors. As far as the economist is concerned, the final outcome is indeterminate—almost as indeterminate as the haggling between two millionaires over the price paid for a rare oil painting.

On this, other theorists can also be cited, e.g., Hicks (1955, p 390). The economic theory of supply and demand could define some limits to what wages can be set, but within those limits closer determination requires that other relationships are introduced.

That was in the 1950s. It is interesting to note in passing that the indeterminacy of wages from theory also characterizes the now standard Diamond-Mortensen-Pissarides (DMP) search and matching models used in academic economics, see e.g., Diamond (1982) and Mortensen and Pissarides (2011). In these models the equilibrium wage is only set-determined, Hall (2005). In order to determine the level of the equilibrium wage, more theory about wage adjustment has to be added to the core search model with a Nash bargaining solution, for example a degree of real wage rigidity, Krogh (2016).

Nevertheless, by the mid 1980s it would have appeared old-fashioned to present a macroeconomic model with exogenous rather than exogenous wage level, see e.g. Wallis et al. (1984). Clearly, something had happened along the way.

For example, at the same time as it was seen as challenging to determine wages theoretically, it was observed that actual wage bargains were struck year after year, and that they were rationalized by considerations of profits, cost of living and relative wages (fairness). These observed regularities, that were documented early by for example Dunlop (1944), gave reason to believe that wage formation could be subject to econometric treatment, for example by the use of multivariate techniques which became increasingly available.

It is interesting to note that the idea to relate wage adjustment to a measure of disequilibrium in the labour market happened early in macroeconomic modelling. Klein (1947b) introduced such a relationship into his IS-LM framework (although Klein (1947a) did not, see De Vroy and Malgrange (2010)). The successful Klein-Goldberger model (1955) included an estimated version of that relationship.. Unemployment was measured in million workers rather than as a percent, but it nevertheless clearly predated the curve of Phillips (1958). Moreover, Klein's and Goldberger's wage equation had the lagged change in the GDP deflator as a second explanatory variable, thus predating also the expectation augmented Phillips Curve Model (PCM below).

Another development was that inflation in the western world after the Second World War years gradually came to undermine the situation with relative price stability simultaneously with full employment. By the late 1970s, inflation was seen as a "blight on the stability and efficient performance of the leading economies and to a potential threat to the preservation of democratic societies", Hirsch and Goldthorpe (1978, p. 1). Hence, is not surprising that during the 1960s and 1970s, studies of the economic mechanisms of inflation was intensified in all western countries. With the exception of monetary models of inflation, these studies identified wage setting as an integral part of the inflation process. When combined with specifications of firms' price setting strategies, the approach resulted in wage and price setting equations that could be grafted into a medium term empirical macro model to become the supply-side of the model, see e.g., Bårdsen et al. (2005). This created an expectation among the users of macroeconometric models that the national wage level was endogenized, and model producers duely responded to that call to action.

The question of whether price stability and full employment could be simultaneously achieved may have motivated much of the econometric literature that may have started with Bill Phillips' 1958 paper, but which soon lost contact with it. Perhaps this happened because Phillips' research question was very clear: His view was that over a long data sample, the relationship that determined the change in money wages was determined by supply and demand, as captured by the rate of unemployment, institutional factors did not go into it. In this he was clearly in opposition to the wage theory of his day, which claimed that many institutional and psychological factors mattered, that the "wage equation" was L-shaped and that the attainment of full employment with relative price stability was possible. None of these views or claims were correct if Phillips was right, Forder (2014, p 31) and Forder (2019).

But Phillips was wrong in his original hypothesis. Soon afterwards, Lipsey (1960) noted that his estimated Phillips curves were different in different periods. Afterwards, the econometric time series modelling of wages seems to have been split in two main currents. The first is the augmented Phillips curves, where price changes are brought into the model, and where there is a distinction between a downward sloping short-run relationship between wage change and unemployment and a vertical long-run Phillips curve. As noted above, these steps had already been tried with success in the Klein and Goldberger model. In 1972, James Tobin summarized the prevailing view of the dynamics of aggregate supply as consisting of a wage Phillips curve and price mark-up equation (specifies in log-differences), Tobin (1972). From this era stems the term *natural rate of unemployment* and its cousin the NAIRU (nonaccelerating inflation rate of unemployment"). An important distinction were between the short-run Phillips curve, where wage setters expectation about the increase in cost of living deviate from inflation, and the long-run Phillips curve where expectations are fulfilled. The natural rate corresponds to the steady state solution of the system when the long-run Phillips curve is vertical Phelps (1967) and Friedman (1968).

For a young person who starts her orientation in macroeconomic modelling in the second decade of the 2000s, it may be difficult to to fully appreciate the formative force that the scientific and practical dimensions of the "price stabilization problem" has had on the development of the field. From this epoch stems the intellectual origin of the coordination by pattern-wage bargaining known as the Norwegian model of inflation (aka *Scandinavian Model*), Aukrust (1977), Gjelsvik et al. (2020).[1]. As late as in 1990, an issue of *Oxford Review of Economic Policy* was titled *Inflation*. In the first sentence in Stephen Nickell's offering was "Inflation is endemic in Britain", Nickell (1993, p. 26). Nickell, in collaboration with among other Richard Layard, developed an original model framework for short and medium term analysis where the supply side of the economy played a central role. The supply side was represented by a pair of wage and price setting equations, albeit in levels not in differences as in the Phillips curve model that prevailed before. One thesis of the so called Layard-Nickell model was that the potential of inflation pressure (due to for example excessive wage demands by unions) would not lead to higher inflation, but to higher unemployment, see Blanchard and Fisher (1989, p. 551-55) and the references therein to Layard and Nickell (1986), Layard et al. (1991) among other contributions.

Despite the levels-in-variables formulation of the Layard-Nickell model it is a natural rate/NAIRU model, in the same way as its Phillips curve predecessor. This is due to the "accelerationist" heuris-

[1]This system of coordination has in recent years been favourably reviewed in OECD surveys of Norway: "The system of collective bargaining based on coordinated annual wage increases works well, providing top-level guidance on wage increases that is anchored in macroeconomic realities",OECD (2019, p.37)

tics of the model, Kolsrud and Nymoen (2015). That means: There is a degree of supply-demand balance in the economy, measured by the rate of unemployment with the property that wage and price inflation increase if the economy tightens and slows down if the economy is slacker. That special state of the real economy is the natural rate of unemployment or the NAIRU, Solow (1999). It is a unique unemployment rate that reconciles the competing real-wage claims of workers and firms and it aligns expectations with the steady-state inflation of the model. The implied dynamics of the Layard-Nickell supply-side model therefore belong to the Phillips curve category. Manning (1993) made the point by presenting an aggregate wage model equation in levels which was under-identified, while the parameters of the dynamic adjustment equation could be estimated consistently.

The category of Phillips-curve models has become extended by the New-Keynesian Phillips curve (NKPCM), see e.g., Clarida et al. (1999), Svensson (2000), (Woodford, 2003, Ch. 3) among others. There are however remarks to this story. In its original derivation and specification, the NKPCM was a model of price inflation with the wage share as a so called forcing variable, Galí and Gertler (1999). Specifically, forcing variable meant that the closed form solution of the NKPCM (structural inflation in Gali and Gertler's terminology) was based on an autoregressive model equation for the wage-share. The degree of imbalance in the labour market did not have any role in the model of Galí and Gertler (1999). And it had no implications about nominal wage changes. It appears that the "Phillips curve" part of the name of the new model class just followed from the habit among economists to call all model equations normalized on wage changes, or price level changes, for Phillips curves.

However, as the new inflation equation was taken up by the producers of the dynamic stochastic general equilibrium (DSGE) macro models that became dominant during the first two decades of the new millennium, adaptations and modifications have produced NKPCs with properties that are of the accelerationist type. First, the same microeconomic theory that was first used to derive the equation with price level change on the left-hand side of the equations and future expected price change on the right hand side was re-applied to wage setting, leading to a wage NKPCM equations, Galí (2011). Second, DSGE model builders have been pragmatic in the choice of forcing variable. The balance of the real economy is often measured by an output-gap variable in the presentations of the models used by inflation targeting central banks. Hence, in model based forward guidance, inflation is shown to approach its target simultaneously with a closing of the output gap. NKPCM model equations nevertheless stand out since they contain lead-variables (forward-looking) as a consequence of their derivation from microeconomic principles. This raises challenges for their inclusion in an empirical macro model. However, in paragraph 5 we discuss strategies for empirical assessment of NKPCMs *qua* empirical model equations for supply side dynamics.

The second branch of the econometric literature can be seen as the a continuation of the L-shaped theory of the 1950s. In these econometric models, it is *possible* but far from certain that price growth can be stabilised at any 'going' rate of unemployment. Hence, we recognise the "exogeniety theory" of wages with respect to unemployment.

It was the British econometrician Denis Sargan who started the second main branch of econometric wage and price modelling, and they were first known as error-correction models, see Sargan (1964,1971,1980). These models were precursors of formal cointegration analysis, Hendry and Phillips (2019). In current parlance they are referred to as equilibrium correction models, ECMs, since they are modelling wages and prices as adjusting towards existing equilibrium relationships, which in turn are interpretable as cointegration relationships (given an $I(1)$ statistical model of the system), Hendry (2003).

Sargan seemed to have what had by the late 1960s become commonly known as the Phillips curve as a reference point when he later revisited his work from 1964. As noted by Ericsson et al. (2001) he wrote that the error correction form arose "by noting that if the lagged real wage is introduced as a variable into a standard wage Phillips curve it is statistically significant. This is enough to reject models which exclude this variable." [2] Hence, the choice between ECM and Phillips curve versions of the wage equation could be made empirically, using a simple t-test. In practice, it may not be quite that straight forward because contesting wage model equations may differ in several other respects. However, in most cases a minimum nesting model of the two can be formulated, and the decision can be made either by encompassing testing or by a variable selection procedure.

This point, about the possibility for empirically based model selection, is important because the two specifications, ECM or Phillips curve form wage setting equations, have different implications for the overall model properties. At this point we only bring up one such property, since we have already mentioned the accelerationist nature of Phillips curve dynamics, and the associated natural rate property. As we make use of below, mathematically, the potential for stable inflation (with zero

[2] Sargan (1971, p. 52).

as a particular special case) is just an implication of modelling wage and price levels as generated by stochastic difference equations, and making that system subject to rank reduction (at the zero (long-run) frequency). As long as that rank (while reduced) is not zero, price change stabilization is vindicated as a theoretical property. As noted above, there is modern economic theories of an indeterminate equilibrium wage level compatible with a set or range of unemployment rates. One example is found in Hahn and Solow (1997) where there no single critical (or natural) level of unemployment at which supply side dynamics is stabilized.

The economic interpretation of wage-price ECM models have developed considerably. The equilibrium correction variable can be formulated as the difference between last period's nominal wage (in logs) and a wage norm. That wage norm can in it's turn be theoretically motivated by models of collective bargaining about nominal wages. When combined with a dynamic price setting equation consistent with monopolistic competition among the firms that operate in the product market, we obtain what is often referred to an incomplete competition model (which has been refereed to as ICM in the literature) of wage and price setting.

One reason why wage-price ECMs may be a a good starting point for empirical wage modelling, is that they can be formulated in such a way that the standard Phillips curve specification become encompassed by the ECM system. In that way, it also provides the econometric framework for testing the Phillips curve, and restrictions that are associated with it, such as vertical long-run Phillips curve restrictions and natural rate of unemployment restrictions.

3 A model typology

We can represent the different models of the supply side that we have covered above, and which still are in use in macroeconomic models, as members of a model typology, Bårdsen et al. (2005, Ch. 7.5.1).

There can be open and closed economy versions of all the models, and to simplify notation we only give the closed economy versions here. Open economy considerations will be introduced in later paragraphs, where we look at empirical implementations of the models.

Let w be the wage level and p the price level (GDP deflator or consumer price index) ; with a as average labour productivity, the wage-share is given as $ws = w - a - p$; u is the unemployment rate;

all measured in logs. We abstract from all other variables that are often used in macroeconomic models (e.g., tax rates, foreign prices, an output-gap variable).

A model of the supply side general enough for the typology then takes the form:

$$\Delta w = \pi_w \Delta p^e + \tau_w \Delta a - \beta_w ws - \sigma_w u, \tag{4}$$

$$\Delta p = \pi_p \Delta p^e + \tau_p (\Delta w - \Delta a) + \beta_p ws, \tag{5}$$

where Δp^e is "expected inflation" and the dynamics is to be specified for each model. Constant terms and random disturbances are omitted for simplicity.

Although the structure is very simple, the different models drop out as special cases:

1. The Phillips Curve Model (PCM):

$$\Delta w_t = \pi_{w1} \Delta p_t - \sigma_{w1} u_t, \tag{6}$$

$$\Delta p_t = \tau_{p1} (\Delta w_t - \Delta a_t). \tag{7}$$

   The first equation the PCM is the wage Phillips curve, hence $0 < \pi_{w1} \leq 1$, $\sigma_{w1} > 0$ and (implicitly) $\beta_{w1} = \tau_w = 0$. If $\pi_{w1} = 1$ the wage PCM is vertical, and the natural rate is determined from that relationship alone (zero here since an intercept is omitted).

2. The wage-price equilibrium correction model (WP-ECM):

$$\Delta w_t = \pi_{w2} \Delta p_t + \tau_{w2} \Delta a_t - \beta_{w2} ws_{t-1} - \sigma_{w2} u_t, \tag{8}$$

$$\Delta p_t = \tau_{p2} (\Delta w_t - \Delta a_t) + \beta_{p2} ws_{t-1}. \tag{9}$$

   In equation (9), the equilibrium, correction is simply the lagged wage-share, which implies that static or (long-run) homogeneity is imposed.

3. The New Keynesian Phillips Curve model (NKPCM) is given as:

$$\Delta w_t = \Delta p_t + \Delta a_t - \beta_{w3} ws_{t-1}, \tag{10}$$

$$\Delta p_t = \pi^f_{p3} \Delta p^e_{t+1} + \pi^b_{p3} \Delta p_{t-1} + \beta_{p3} ws_t, \tag{11}$$

where the expectations term Δp^e_{t+1} in (11) is to be modelled as a conditional expectation (i.e., the rational expectations hypothesis), see Section 5. We follow custom and refer to the case of $\pi^b_{p3} = 0$ as the (theoretically) pure New Keynesian Phillips curve, while $\pi^f_{p3} > 0$ and $\pi^b_{p3} > 0$ define the hybrid form of the NKPCM. As noted above the NKPCM has become an integral part of DSGE models.

Of course, there exist a host of other, more elaborate versions of each model, perhaps with richer dynamics as the most obvious. However, the purpose here is to highlight the demarcation lines between the model categories. One difference between the old and the new Phillips curve model, is that the rate of unemployment appears in the PCM but not in the NKPCM. However, that gap can can be bridged by adapting the theory of staggered nominal adjustment to wages as well as to prices, leading to the New Keynesian *wage* Phillips curve, cf. Erceg et al. (2000), Galí (2011).

The real demarcation line between the old and the new Phillips curve models is that the lead-variable Δp^e_{t+1} in the NKPCM is represented by the hypothesis of of rational expectations.

The wage-price ECM mainly differs from the NKPCM in the treatment of expectations and from the PCM in the latter's exclusion of equilibrium correction mechanisms that follow from the theories of nominal and real wage formation integrate rent-sharing, collective wage bargaining and monopolistic price setting.

As hinted above, it is relatively straight forward to formulate a joint multiple-equation system for the WP-ECM and the PCM. The exact definition of Δp^e_{t+1} as the mathematical conditional expectation of Δp_{t+1} in the NKPCM makes, it necessary with an extra steps before we can enlarge the common ground to include tht model. We therefore treat WP-ECMs and PCMs in the next section, and return to the NKPCM in Section 5.

4 WP-ECM and PCM models of the supply side

In this section, an equation system that can represent a model of the supply side of an open economy macro model is analysed. Extension to the open economy does not interfere with the model typology while it is important for the relevance of the framework to operational models.

4.1 Theory

We want to keep the framework simple and only introduce two new variables. They are the domestic producer price, q, and the import price index denoted in domestic currency, pi. As a consequence, we re-define the symbol p to be the consumer price index, which represents a weighted average of q and pi.

The following presentation follows Kolsrud and Nymoen (2014) closely, however the framework has been developed in stages and applied to different data sets, see e.g., Kolsrud and Nymoen (1998), Bårdsen and Nymoen (2003,2009), Akram and Nymoen (2009), Bårdsen et al. (2012).

a) Nominal and real trends

We begin by defining the exogenous stochastic trend variables of the model. Stochastic trends are represented by integrated variables, $I(d)$, where d denotes the order of integration. $I(0)$ is used to denote stationarity, and $I(1)$ denotes a time series variable with a unit root (at the long run frequency of the series).

There are two exogenous $I(1)$ variables in the model: one nominal trend and one real trend. The nominal trend is the price of imports pi in domestic currency. We write the equation as a random-walk with a positive drift term, g_{pi}:

$$pi_t = g_{pi} + pi_{t-1} + \varepsilon_{pit},\ g_{pi} > 0 \text{ and } \varepsilon_{pit} \sim N(0, \sigma_{pi}^2). \tag{12}$$

The drift parameter g_{pi} represents underlying foreign inflation. The disturbance term ε_{pit} may include international price shocks or a stationary nominal foreign currency exchange rate (normalised to zero mean).

We can think of pi_t as the sum of the logarithms of a price index denoted in foreign currency and a nominal exchange rate index. Hence, the parameters of (12)are likely to depend on the exchange rate regime. However, under the maintained assumption that foreign currency denoted price index is $I(1)$, it follows that pi_t is also $I(1)$ in both a floating and a fixed exchange rate regime (even if the nominal exchange rate should be $I(0)$ in one of them).

Having introduced a price index of imports, we can define the logarithm of the consumer price index, p_t:

$$p_t = \phi q_t + (1 - \phi) pi_t, \tag{13}$$

where the parameter $0 < \phi < 1$ measures the share of imports in total consumption. (13) is only a stylized definition of the consumer price index, but it allows us to make the distinction between the consumer real wage $(w - p)$ and the producer real wage $(w - q)$, which is important in models of wage formation.

We assume that there is secular growth in labour productivity. Hence, the logarithm of labour productivity a_t is also specified as a random-walk variable with positive drift:

$$a_t = g_a + a_{t-1} + \varepsilon_{at},\ g_a > 0 \text{ and } \varepsilon_{at} \sim N(0, \sigma_a^2). \tag{14}$$

We model domestic wage and price dynamics as conditional on pi_t and a_t. Equations (12) and (14) will therefore imply that q_t, the (log of the) price level on domestic products, and w_t, the (log of) wage compensation per hour, will be non-stationary variables. In the case where the domestic wage and price setting system is dynamically stable, q_t and w_t will be integrated of order one, $I(1)$. If the domestic wage-price spiral is unstable, domestic wages and prices become "more non-stationary" than their foreign counterparts, theoretically they may become $I(2)$, or there may be 'wage and price bubbles' (technically due to explosive characteristics roots).

b) Wage-price spiral

We next define two theoretical (latent) real wage variables: The targeted producer real wage from the point of view of the firms, rw_t^f, and the planned or expected bargained producer real wage, rw_t^b. They are given by the following two equations:

$$rw_t^f = w_t - q_t^f = -m_q \qquad + \ a_t + \vartheta\, u_t,\ \ \vartheta \geq 0 \tag{15}$$

$$rw_t^b \equiv w_t^b - q_t = \quad m_w + \omega\,(p_t - q_t) + \iota\, a_t - \varpi\, u_t, \tag{16}$$

with $\iota > 0$, $0 \leq \omega \leq 1$, $\varpi \geq 0$, see Nymoen and Rødseth (2003).

Another term used is *wage norm*, which captures the idea of a system where in particular collective bargaining plays a role in defining a norm for the actual wage. At an given month, quarter, or year, the actual wage level will typically deviate from the norm, but at the same time it will attracted by the norm (i.e., equilibrium correction)

Equations (15) and (16) can be drawn as a "price curve" and a "wage curve" respectively in a diagram with real wage (or wage share) on the vertical axis and u_t on the horizontal axis. In the case of $\vartheta = 0$, the price curve becomes horizontal.[3]

Care must be taken though: The intersection of the curves does *not* represent the determination of the rate of unemployment, in the NAIRU meaning. This is because we yet have an under-determined model, with more variables than equations.

Returning to the price-setting equation (15), q_t^f denotes the price level set by the firm on basis of expected nominal marginal labour costs $w_t - a_t$.[4] In (16), w_t^b denotes the planned bargained nominal wage, given the expectations of the two price level indices q_t and p_t. A reasonable assumption is that the elasticity ι in (16) with respect to productivity is close to unity, as in Nymoen and Rødseth (2003). The standard assumption about the sign of the coefficient for unemployment, ϖ, is that it is non-negative, hence $-\varpi < 0$, as indicated. The coefficient ω is called the wedge coefficient since it is multiplied by $(p - q)_t$ which is the wedge between consumer and producer real wages (we abstract from tax rates). The wedge coefficient is assumed to be non-negative, $\omega \geq 0$, see Rødseth (2000, Ch. 8.5).

rw_t^f and rw_t^b are $I(1)$ variables by construction and can be *co-integrated* with the producer real-wage rw_t. With that in mind we define two variables ecm_t^f and ecm_t^b:

$$ecm_t^f \equiv rw_t - rw_t^f = q_t^f - q = w_t - q_t - a_t - \vartheta\, u_t + m_q \tag{17}$$

$$ecm_t^b \equiv rw_t - rw_t^b = w_t - w_t^b = w_t - q_t - \iota\, a_t - \omega(1-\phi)re_t + \varpi\, u_t - m_w, \tag{18}$$

where we have used (13) to write the wedge variable as

$$(p - q)_t = (1 - \phi)(pi - q)_t,$$

[3]Referred to as normal cost pricing. This hypothesis states that any procyclical fluctuations in the mark-up of prices over actual unit costs are merely side effects of fluctuations in productivity, cf. Barker and Peterson (1987, Ch. 13.5). If $(p_t - q_t)$ changes, the wage curve shifts if $\omega > 0$.

[4]We do not introduce explicit notation for firms' expected wage, because with zero mean $I(0)$ expectation errors, it will not have any implications for co-integration (or not) between the variables. But it is understood that w_t in (15) is an expected variable, and that p and q in (16) likewise denote expected prices in this context.

and then defining the *real exchange rate* $re_t \equiv pi_t - q_t$. The point of this step will soon become clear, since the model can be solved in terms of re_t and one other real variable called the wage share. We may note, in passing, that if cointegration does hold, both ecm_t^f and ecm_t^b are $I(0)$ variables. But we do not impose that from the outset.

For short-run dynamics, the wage-price spiral, we use the simultaneous equations model:

$$\Delta q_t = c_q + \psi_{qw}\, \Delta w_t + \psi_{qpi}\, \Delta pi_t - \varsigma\, u_{t-1} + \theta_q ecm_{t-1}^f + \varepsilon_{qt}, \tag{19}$$

$$\Delta w_t = c_w + \psi_{wq}\, \Delta q_t + \psi_{wp}\, \Delta p_t - \varphi\, u_{t-1} - \theta_w ecm_{t-1}^b + \varepsilon_{wt}, \tag{20}$$

where Δ is the difference operator, $\Delta q_t \equiv q_t - q_{t-1}$, and where ψ_{qw}, ψ_{qpi}, ψ_{wq}, ψ_{wp}, ς, φ, θ_q, $\theta_w \geq 0$, $\varepsilon_{qt} \sim N(0, \sigma_q^2), \varepsilon_{wt} \sim N(0, \sigma_w^2)$.[5]

At first it may seem cumbersome that u_{t-1} is included in both (19) and (20), since it is already included in ecm_t^f and ecm_t^b. But this is done to extend the framework to the case where cointegration fails in the sense that ecm_t^f and/or ecm_t^b may be $I(1)$, instead of $I(0)$. For example, $ecm_t^b \sim I(1)$ $\Longrightarrow \theta_w = 0$, but $\varphi > 0$ is still a logical possibility as long as $u_t \sim I(0)$, which is seen to give a wage Phillips curve model(wage PCM). Conversely, $ecm_t^b \sim I(0) \Longrightarrow \theta_w > 0$ since an equilibrium correction model (ECM) is implied by the Granger-Engle (1987) representation theorem. But in that case, $\varphi = 0$ is the only logically consistent possibility.

Substituting for the two *ecms* in (17) and (18), we get:

$$\begin{aligned} \Delta q_t &= (c_q + \theta_q\, m_q) + \psi_{qw}\, \Delta w_t + \psi_{qpi}\, \Delta pi_t - \mu_q\, u_{t-1} \\ &\quad + \theta_q\, (w_{t-1} - q_{t-1} - a_{t-1}) + \varepsilon_{q,t}, \end{aligned} \tag{21}$$

$$\begin{aligned} \Delta w_t &= (c_w + \theta_w\, m_w) + \psi_{wq}\, \Delta q_t + \psi_{wp}\, \Delta p_t - \mu_w\, u_{t-1} \\ &\quad - \theta_w\, (w_{t-1} - q_{t-1} - \iota\, a_{t-1}) + \theta_w\, \omega\, (p_{t-1} - q_{t-1}) + \varepsilon_{w,t}, \end{aligned} \tag{22}$$

where the notation $\mu_q = \theta_q\, \vartheta + \varsigma$ and $\mu_w = \theta_w\, \varpi + \varphi$ are used to nest PCM and WP-ECM specifications in the same formulation, as just noted.

Although the term equilibrium correction, and error correction (its forerunner), stem from dy-

[5] For coefficients ψ_{wq}, ψ_{qw} and ψ_{wp}, ψ_{qpi}, the non-negative signs are standard assumptions. Negative values of θ_w and θ_q, can give rise to explosive dynamics in wages and prices (hyperinflation), which is different from the low to moderately high inflation scenario that we have in mind for this paper.

namic econometrics, the concepts has clear economic connotation. The point was expressed by Sargan (1971, p 52), who saw "error correction" variables as natural to use in a model of bilateral monopoly, and stating in a later paper that "clearly both sides in a wage bargaining procedure are concerned with the real wage", see Sargan (1980, p 98).[6] In our notation this is implied by $\theta_w > 0$.

As pointed out above, the understanding that nominal wage setting was a result of a genuine bargaining process with power relationships, was quite common in the 1960s and 1970s. Another econometrician, Trygve Haavelmo, presented the same line of thought in some lectures on inflation in the mid 1970s, see Anundsen et al. (2012). The label 'Conflict models' were put on these models at the time, but a better name might have been 'Compromise models'. Haavelmo in particular, was clear that if the model solution was stable, the equilibrium real wage represented a compromise, like a "quasi peace".

In the same style, the term Incomplete Competition Model, ICM, has been used, Bårdsen et al. (2005, Ch. 5). The relevance of incomplete competition in the product market is mainly to draw the distinction between perfect competition and monopolistic competition. In the labour market, the label incomplete competition is even more to the point, since limitation of *unwanted* competition is a prerequisite for well functioning labour market regulation based on collective agreements. The emphasis is on "unwanted" because it is clear that unless there is relatively deep consensus about how much competition is wanted, or "right", in the labour market, a system of collective bargaining will probably become gradually undermined or some suddenly be disrupted by political and industrial strife, Nymoen (2017).

The alternative to the WP-ECM is the wage Phillips curve, defined by $\theta_w = 0$, which is relevant if supply and demand conditions, after all, are the only really wage determining factors, see Bårdsen et al. (2005, Ch.3-6). In order to distinguish between WP-ECM and PCM specifications we therefore have:

$$\text{WP-ECM: } \theta_w, \theta_q > 0 \text{ and } \varsigma = \varphi = 0 \Longrightarrow \mu_w = \theta_w \varpi \text{ and } \mu_q = \theta_q \vartheta$$

$$\text{PCM: } \theta_w = \theta_q = 0 \text{ and } \varsigma = \varphi > 0 \Longrightarrow \mu_w = \varphi \text{ and } \mu_q = \varsigma$$

[6] Later theoretical derivations in the literature, using the Nash-solution, agree that $\theta_w > 0$ is implied by collective bargaining, but also find $\omega = 0$ to be equally theoretically meaningful. It has even been argued that in a theoretically pure wage bargaining model the compensation for cost-of-living increases should be purged from the short-run dynamics, Forslund et al. (2008).

Note that the WP-ECM is consistent with a horizontal "price curve", since $\vartheta = 0 \Longrightarrow \mu_q = 0$, but not $\theta_q = 0$.

c) VAR formulation

Using the differenced version of the definitional (13):

$$\Delta p_t = \phi \, \Delta q_t + (1 - \phi) \, \Delta pi_t \tag{23}$$

in combination with (21) and (22), the model can be re-formulated as a (open) VAR in the two variables re_t and $ws_t = w_t - q_t - a_t$, the logarithm of the wage share. If the parameter restriction $\iota = 1$ on a_{t-1} (in wage-setting) is imposed, the VAR for re_t, ws_t becomes independent of the labour productivity level, but depend the growth rate, Δa_t. As long as a_t is specified as a random walk with drift, this step is mainly to save notation.[7]

The two-variable VAR with re_t, ws_t as endogenous variables is conditional on u_{t-1} (and Δpi_t and Δa_t). It is of interest since it can be used to answer one of the classical questions in this area, namely whether is logically possible for wage-price dynamics to be stable for a given rate of unemployment, not only for a natural rate (i.e., "u-star"). This question was analysed by Kolsrud and Nymoen (1998) who formulated a set of sufficient conditions for (global asymptotical) stability:

$$\theta_w > 0 \quad \text{and} \quad \theta_q > 0 \quad \text{and} \quad \omega > 0 \quad \text{and} < 1 \quad \varphi_{qw} < 1 \tag{24}$$

The first two conditions represent equilibrium correction of wages (as noted) and prices with respect to their targets. The third condition states that there is a long-run wedge effect in nominal wage-setting. Finally a particular form of dynamic homogeneity is precluded by the fourth condition: for stability a one point increase in the rate of wage growth must lead to less than one point increase in the rate of price growth. Note that $\varphi_{qw} < 1$ is different from (and more restrictive) than dynamic

[7] An alternative is to define a productivity corrected producer real wage: $rwc_t = w_t - q_t - \iota a_t$. This may be needed in empirical models, where the $\iota = 1$ cannot be imposed after testing.

homogeneity in general which would entail

$$\psi_{qw} + \psi_{qpi} = 1 \tag{25}$$

$$\psi_{wq} + \psi_{wp} = 1. \tag{26}$$

Dynamic homogeneity, in this usual sense, is consistent with a stable steady state of the WP-ECM at a given rate of unemployment. Hence, every unemployment rate is a "u-star". It is an interesting result, since in the PCM, dynamic homogeneity is exactly what defines a vertical Phillips curve and hence a unique natural rate "u-star".

In econometric terms, the above analysis is based on an assumption about strong exogeneity of u_t, i.e., no feed-back from the real wage of the real exchange rate on the unemployment rate. In the context of modelling the total economy, this is unsatisfactory. In order to allow for joint feed-back effects we can formulate the following (closed) VAR with three endogenous variables:

$$\underbrace{\begin{pmatrix} re_t \\ ws_t \\ u_t \end{pmatrix}}_{\mathbf{y}_t} = \underbrace{\begin{pmatrix} l & -k & n \\ \lambda & \kappa & -\eta \\ -\rho & \varrho & \alpha \end{pmatrix}}_{\mathbf{R}} \underbrace{\begin{pmatrix} re_{t-1} \\ ws_{t-1} \\ u_{t-1} \end{pmatrix}}_{\mathbf{y}_{t-1}} + \underbrace{\begin{pmatrix} e & 0 & -d \\ \xi & -1 & \delta \\ 0 & 0 & c_u \end{pmatrix}}_{\mathbf{P}} \underbrace{\begin{pmatrix} \Delta pi_t \\ \Delta a_t \\ 1 \end{pmatrix}}_{\mathbf{x}_t} + \underbrace{\begin{pmatrix} \epsilon_{re,t} \\ \epsilon_{ws,t} \\ \epsilon_{u,t} \end{pmatrix}}_{\epsilon_t} \tag{27}$$

The third row of (27) contains a simple dynamic relationship between u_t, the rate of unemployment in period t and the lagged re_t and ws_t. Since the two first rows represent the reduced form of the model of the supply-side, we can interpret the last row as a stylized dynamic aggregate demand relationship. Since increased re_t means improved competitiveness (real depreciation) it is reasonable that $\rho \geq 0$. For the coefficient of the lagged real wage the default may also be to set $\varrho \geq$, although real wages as noted, influence employment and labour supply though several channels, some of them with opposite signs.

For the autoregressive parameter we set $0 < \alpha < 1$, which implies persistence of unemployment, but within the limits of stationarity. The VAR disturbance $\epsilon_{u,t}$ contains all other variables that might affect u_t.

For the WP-ECM, the **R** and **P** coefficients associated with re_t can be shown to be:

$$
\begin{aligned}
l &= 1 - \theta_w \, \omega \, \psi_{qw} \, (1 - \phi)/\chi, \\
k &= (\theta_q - \theta_w \psi_{qw})/\chi, \\
n &= (\mu_q + \mu_w \, \psi_{qw})/\chi, \\
e &= 1 - (\psi_{qpi} + \psi_{qw} \, \psi_{wp} \, (1 - \phi))/\chi, \quad = 0 \text{ if dynamic homogeneity} \\
d &= (m_q \, \theta_q + c_q + (m_w \, \theta_w + c_w) \, \psi_{qw})/\chi,
\end{aligned}
$$

where the denominator is: $\chi = 1 - \psi_{qw}(\phi\psi_{wp} + \psi_{wq}) > 0$. For ws_t the coefficients in **R** and **P**, in the case of WP-ICM, are:

$$
\begin{aligned}
\lambda &= \theta_w \, \omega \, (1 - \psi_{qw})(1 - \phi)/\chi, \\
\kappa &= 1 - (\theta_w \, (1 - \psi_{qw}) + \theta_q \, (1 - \psi_{wq} - \phi \, \psi_{wp}))/\chi, \\
\eta &= (\mu_w \, (1 - \psi_{qw}) - \mu_q \, (1 - \psi_{wq} - \phi \, \psi_{wp}))/\chi, \\
\xi &= (\psi_{wp} \, (1 - \psi_{qw})(1 - \phi) - \psi_{qpi} \, (1 - \psi_{wq} - \phi \, \psi_{wp}))/\chi, \qquad = 0 \text{ if dynamic homogeneity} \\
\delta &= ((m_w \, \theta_w + c_w)(1 - \psi_{qw}) - (m_q \, \theta_q + c_q)(1 - \psi_{wq} - \phi \, \psi_{wp}))/\chi.
\end{aligned}
$$

These coefficients are non-negative for reasonable values of the structural coefficients. The exception is δ which can be both positive and negative, see Kolsrud and Nymoen (2014).

d) Stability and steady-state unemployment

The condition is that the three eigenvalues of **R** have modulus inside the unit-circle. Even though the theory model is kept simple and transparent, Kolsrud and Nymoen show that the general analytic expressions for the eigenvalues of **R** are too large and complex to be of much help. However, simulation of a numerical version of the theoretical model gives insight, as shown below.

One thing that can be established from the general expression though, is that the dynamic homogeneity restrictions (25) and (26) have no direct implication for dynamic stability. However if the system *is* stable, dynamic homogeneity implies that the steady states of re_t, ws_t and u_t do not depend on the nominal growth rate.

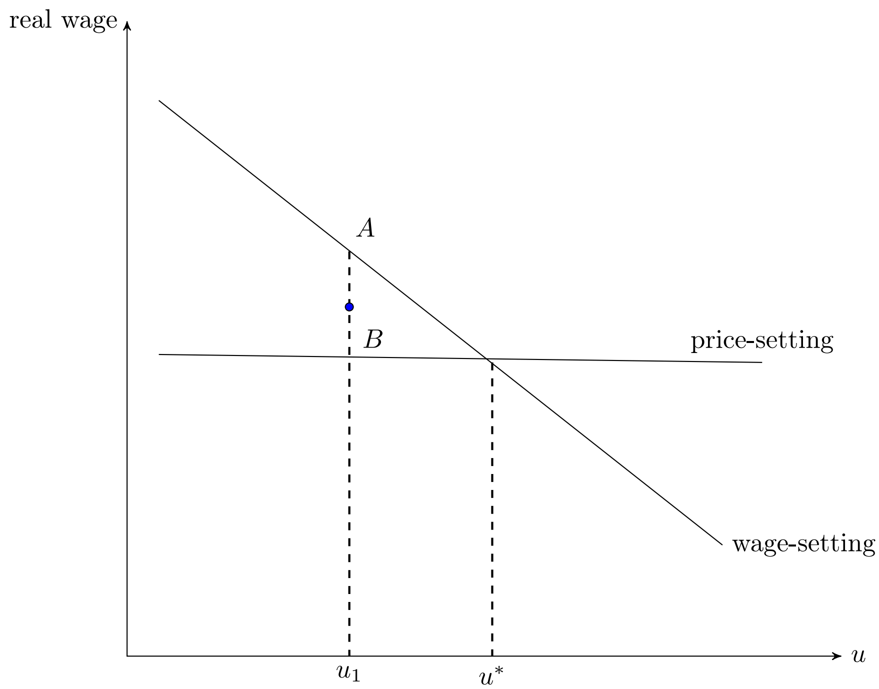

Figure 1: Real wage and unemployment determination. NAIRU (u^*) in the figure and another steady-state rate of unemployment u_1

In the PCM case, $\theta_w = \theta_q = 0$, $\mathbf{R}$ simplifies to

$$\mathbf{R}_{PCM} = \begin{pmatrix} 1 & 0 & n \\ 0 & 1 & -\eta \\ -\rho & \varrho & \alpha \end{pmatrix}$$

and this matrix has one unit-root. This shows that the PCM version of the model is less "inherently stable" than the ECM. Intuitively, if the price and wage equations become disconnected from the profitability and real wage levels, other variables in the system must take over their role in the stabilization. However, that may not be enough to stabilize the system. Hence, since the matrix $(R - I)$ has rank 2 for the PCM, there will in general be stochastic trends in all three endogenous variables. However, there are two implied cointegration relationships between the I(1) variables.

In the real world, there can of course be checks on unstable processes. Specifically, there may be the corrective mechanisms in the form of deflationary (or inflationary) economic policies that Nickell (1990) had in mind for his "inflation is endemic in the UK" paper cited above.

Figure 1 with the lines representing the "wage curve" and the "price curve" mentioned above, represents some of the possibilities. In the model we have formulated, the case of a singular steady-state value of unemployment (the "u-star") consistent with constant inflation may be specific special case. It is represented by u^* in the figure, and consistent with Layard's and Nickell's thesis:

> 'Only if the real wage (W/P) desired by wage-setters is the same as that desired by price setters will inflation be stable. *And, the variable that brings about this consistency is the level of unemployment'*.[8]

However, in the model, there are other possible solutions. The NAIRU u^* is given by the intersection of the curves, but another steady-state rate of unemployment u_1 may be lower than u^* , the case shown in the graph, or higher. The figure further indicates (by a •) an equilibrium wage share at a point on the line segment A-B: Heuristically, this is a point where price setters are trying to attain a lower real wage by nominal price increases, at the same time at the wage bargain is delivering nominal wage increases that push real wage upwards, it is a tug-of-war equilibrium.

Another interpretation of Figure 1 is that u^* represents an initial steady state situation (where the economy has been "at rest" for some time), and u_1 represents a new pre-determined steady state after a shock. In this interpretation, there must be a dynamic process that connects the two steady-states. One possibility is that the wage-setting curve drifts away from its initial position, finally reaching its new stationary position, close to B, after an adjustment period, so that u_1 becomes the new steady-state rate of unemployment (or NAIRU if one sticks that terminology).

The model-dependency of the equilibrium (steady-state) rate of unemployment can also be shown formally in our model framework, as special cases that are used in the literature.

For example, simplify the demand side by setting $\rho = 0$, meaning that unemployment only depends on the real wage (an empirical example is given below). The wage Phillips curve version of the model drops out as a special case by imposing the restrictions $\theta_w = 0$ (nominal wage setting) and $\theta_q = 0$, $\varsigma = 0$ (price adjustment). If the system is dynamically stable, the steady state rate of unemployment becomes:[9]

$$u^{\text{WPCM}} = \frac{c_w - g_a}{\varphi} + \frac{(\psi_{wq} + \psi_{wp} - 1)}{\varphi} g_{pi}. \tag{28}$$

[8]Layard et al. (1994, p 18), authors' italics.

[9]Under these assumptions the system separates into a stationary subsystem of u_t and ws_t and a separate random walk process for re_t.

Subject to a dynamic homogeneity restriction: $\psi_{wq} + \psi_{wp} = 1$, the steady-state rate is independent of "imported inflation" g_{pi}:

$$u^{\text{WPCMr}} = \frac{c_w - g_a}{\varphi}, \tag{29}$$

and defines a natural rate of unemployment within the above framework. Even though this may be the expression that most economists would write down when asked, it is nevertheless a special case, and not natural in the sense of being a common steady state value to find, in a dynamics wage price system that allows for more direct equilibrium correction that the adjustment that "goes through" unemployment. It is of considerable interest, as a specifically *restricted* form of Phillips curve dynamics associated with a vertical long-run Phillips curve, central to the accelerationist conceptualization.

If we instead set $\theta_w > 0$, and keep the other simplifying assumptions that led to (29), the expression for the steady state rate of unemployment becomes:

$$u^{\text{WECM}} = \frac{\theta_w c_u - g_a + (\psi_{wq} + \psi_{wp} - 1) g_{pi}}{\theta_w (1 - \rho) + \varpi \varrho}, \tag{30}$$

the superscript WECM indicates that in this special case it is only wage formation that equilibrium corrects around the target (hence $\theta_q = 0$ as noted).

Clearly, the two steady states will be different in general. In may be noted in particular that u^{WECM} depends on c_u which is parameter from the demand side of the model. A structural change in that parameter will affect u^{WECM} while u^{WPCM} is invariant to such a structural break.

e) Numerical simulation of the theory model

To round off the theoretical section, we simulate the theory model for a specific WP-ECM calibration. This will demonstrate dynamic properties of the system, when it is stable.

It is easiest to calibrate the structural form. In the system of equations below, (31) corresponds

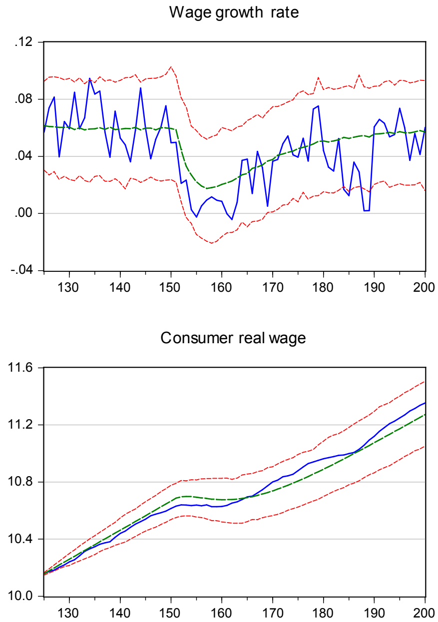

Figure 2: Solution paths for endogenous variables shown in graphs with dashed lines, together with ‘actuals’ (the computer generated time series) and 95 percent uncertainty intervals. The first solution period is period 125 and the final period of the dynamic simulation is number 200.

to (21), and (32) to (22) and so on:

$$\Delta q_t = c_q^c + 0.6\,\Delta w_t + 0.2\,\Delta pi_t + 0.12\,(w-q-a)_{t-1} + \varepsilon_{q,t}, \tag{31}$$

$$\Delta w_t = c_w^c + 0.15\,\Delta q_t + 0.7\,\Delta p_t - 0.15 \cdot 0.35\,u_{t-1} \tag{32}$$

$$- \; 0.15(w-q-1\,a)_{t-1} + 0.15 \cdot 0.8\;(p_{t-1} - q_{t-1}) + \varepsilon_{w,t},$$

$$u_t = c_u^c + 0.25u_{t-1} - 0.25(pi-q)_{t-1} + \varepsilon_{u,t} \tag{33}$$

$$\Delta p_t = 0.5\,\Delta q_t + 0.5\Delta pi_t, \tag{34}$$

$$pi_t = pi_{t-1} + 0.04 + \varepsilon_{u,t}, \tag{35}$$

$$a_t = a_{t-1} + 0.02 + \varepsilon_{a,t}. \tag{36}$$

Note that θ_w has been set to 0.15 and θ_q to 0.12 in order to capture the essence of the incomplete model, and given that, we have set $\varsigma = \varphi = 0$. The numbers chosen for the remaining intercepts c_q^c and c_w^c only influence the means (if they exists) of the simulations. c_u^c is calibrated with a large structural break in period 150, to simulate a permanent change in the mean of u.

In the simulations, the artificial time series of the observable economic variables q_t, w_t, u_t, p_t, pi_t and a_t are generated by using the VAR representation of the complete system (31)-(36). We use the computer generated data to estimate the simultaneous equations model with the use of FIML. Finally, we use dynamic simulation of the estimated structural model and plot the solutions paths of the endogenous variables.

The first row of Figure 2 shows the graphs of the rates of change in the wage in the consumer price index in the simulation. Both wage and price inflation are stable at the start of the period. The reduction of nominal growth rates that appears in period 151 is caused by the structural break in the rate of unemployment in period 150. Since that shift is permanent, the rate of unemployment does not return to the initial low level, as shown in the graph for unemployment in the second row in the figure. However, the reductions in wage and price inflation are temporary, which is a typical trait of WP-ECM, in contrast to the accelerationist PCM. The consumer real wage, shown in the panel to the left in the second row, is also affected by the increase in unemployment. This is because, in the theory model, it is nominal wage growth which is directly 'hit' by the increased unemployment. CPI inflation is reduced, but as a reaction to the moderation in wage growth. Hence, the growth in the real consumer wage is practically brought to a halt by the rise in unemployment, before it gradually 'finds its way back' to the secular productivity driven trend growth of the calibrated model. As the graph shows.

Figure 3 shows the same graphs as Figure 2 in the second row, but in the first row, we have the relative import price (the real-exchange rate re) and the wage-share. In the chosen calibration, and abstracting from the break in u_t, both these variables are stationary, as indicated by the solutions for the period leading up to the increase in unemployment. After the break, the relative price of imports is permanently increased towards a new equilibrium level (a permanent real depreciation). The wage share, even though it is markedly reduced as a consequence of step-increase in u_t, slowly equilibrium corrects back to the pre-break mean.

Hence, although the unions have influence over the money-wage bargains in the model and have a (consumer) real wage target, the simulations show that they do not really control the dynamics and the long run level of the real wage. If that was the case, we would expect to see that the wage share became lowered by the step-up in the unemployment rate.

The insight that the real wage may be only weakly linked to the nominal wage setting is an

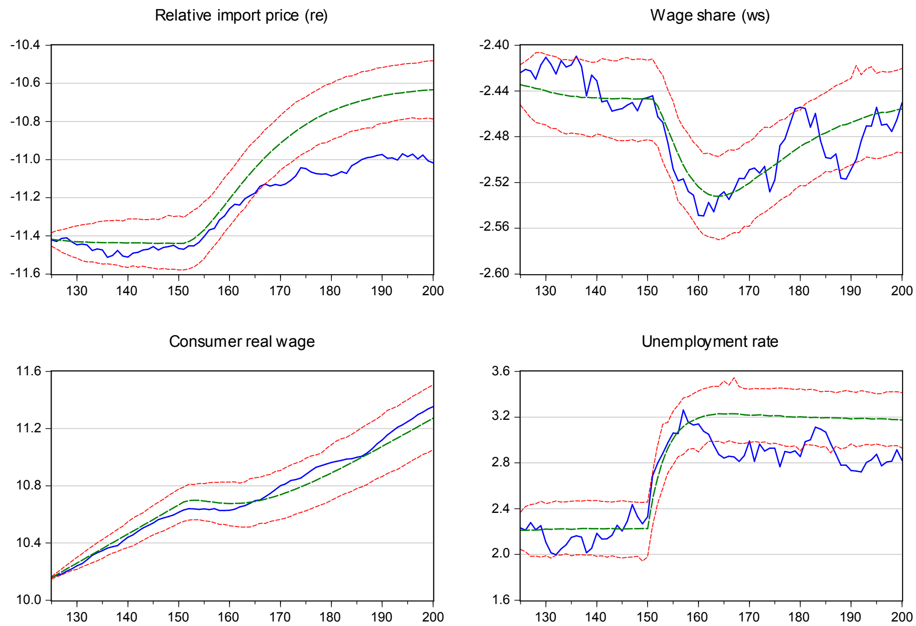

Figure 3: Solution paths for endogenous variables shown in graphs with dashed lines, together with 'actuals' (the computer generated time series) and 95 percent uncertainty intervals. The first solution period is period 125 and the final period of the dynamic simulation is number 200.

old one in macro economics. Keynes was clear about this, see Keynes (1936, p. 12-13). In terms of econometric concepts, the dynamics shown in the figure demonstrates endogenous *co-breaking*, Clements and Hendry (1999, Ch. 9): there are "permanent large-shifts" in the real exchange rate and in unemployment, but there is no break in the unconditional mean of the equally endogenous wage-share, even if the nominal wage in particular is directly affected by unemployment.

Finally, we note that the nominal growth rates are stable (and the same) on each side of the break in the unemployment mean. Clearly, in this model, a unique steady-state rate of unemployment does not follow from constant wage and price inflation rates, as it will do in a PCM of the natural rate (u-star) type.

However, it should be remembered that by looking at only one calibration we may underestimate the complex dynamics that the framework can generate. For example, Kolsrud and Nymoen (2014) show by analysis and simulations that the interplay between parameters can give rise to very different dynamics, some with cycles, other with more smooth stabilization after a shock. This is also echoing Sargan (1980, p 108), who noted how "critically dependent" the dynamics of his wage and price model were on the estimated coefficients in the wage and price equations.

f) Implications for impulse responses, dynamic multipliers and forecasts

The result that the dynamic solution and the eventual steady-state of WP-ECMs and PCMs of the supply side are different, has consequence for other model properties.

Impulse response functions have the same properties as the homogenous part of the solution equations for the endogenous variables. Hence if the solutions are model dependent, the impulse responses will also be different between models.

Dynamic multipliers (derivatives with respect to non-modelled observed variables) become dominated by the homogenous solution after the impact multiplier and a first couple of dynamic multipliers. Hence, they will also differ, also in the case where the specification that (distributed lag part) is the same across models.

Finally, model based dynamic forecasts also become dominated by the homogenous solution (a few periods into the forecast horizon, hence they will be different for WP-ECM and PCM models of the supply-side.

4.2 Empirical illustrations and applications

a) US wage-price dynamics

Bårdsen and Nymoen (2009b) specify and estimate a PCM for annual US data (1962-2004). It is custom to regard this model as the standard North-American model of the natural rate of unemployment. cf. Blanchard and Katz (1999) who argue that a wage equation on equilibrium correction form changes the dynamics fundamentally and can be seen as typical of Europe.

However, as already noted exactly how consequential the difference in nominal wage formation become for the total model properties depends on the strength of equilibrium correction elsewhere in the model. We refer to this phenomenon as *extended equilibrium correction*, since it shows that the issue about mean reverting behaviour of the rate of unemployment is just as much a question about equilibrium correction elsewhere in the system as in the wage equation.

In terms of the parameters above, the PCM specified by Bårdsen and Nymoen had $\theta_q = \theta_w = 0$, while the two coefficients of unemployment in the wage and price Phillips curves (ς and φ) are freely estimated. They then specified an ECM model, with θ_w is freely estimated, but where the price equation is identical to the PCM version (hence $\theta_q = 0$).

In the specification of the W-ECM they also discovered empirically that $\omega = 1$ in the wage-target equation. This implies that the real wage that defines the wage equilibrium correction term is the consumer real wage $w - p$. They also found it indefensible statistically to impose the restriction $\iota = 1$ on the productivity coefficient, and estimated it to be less than 0.5 (but still significantly different from zero).

In the unemployment equation, both the producer real wage (with coefficient ϱ above) and the real exchange rate (with coefficient ρ) enter significantly.

Bårdsen and Nymoen show the solutions of the empirical W-ECM and the PCM. which turn out to be nearly identical. However, this paradox is resolved by remembering the appearance of extended equilibrium correction effects in the two models, which dominates the effects of the different specifications of the wage equation.

In order to specify a PCM which behaves distinctively different from the W-ECM, and more in line with the textbook case of a vertical long run Phillips curve, the equilibrating mechanisms of the model must be restricted much more, hence they considered a *restricted* econometric PCM where there are no extended equilibrium correction. Figure 4 shows how the three different models respond to a permanent and exogenous shock to unemployment. Thus we consider a counterfactual experiment which corresponds to a reduction in the parameter c_u in the third row of the VAR in section 4.1. The shock has been calibrated to correspond to a reduction from 5% to 4.5% in the unemployment rate. In Figure 4, panel a), the graph for the econometric PCM shows the most vigorous wage response, corresponding to a lowering of the annual rate from 5% to 4.4% in the third year after the shock. There is less marked difference between the responses of the W-ECM and the PCM in panel b), which shows the inflation response, which is due to the direct effect of the rate of unemployment in the Δp_t equation of both models.

The differences between the three models are also apparent in panels c) and d), showing the cumulated multipliers for unemployment and in the wage share. For the W-ECM and the PCM, there is a sharp and lasting increase in the rate of unemployment. This kind of response cannot be reconciled with the stylized Phillips curve model, which only allows shocks that arise in the Phillips curve equation to affect the steady state unemployment rate. Nevertheless, the responses in Figure 4 happen for perfectly logical reasons since the empirical W-ECM and PCM are in fact quite similar

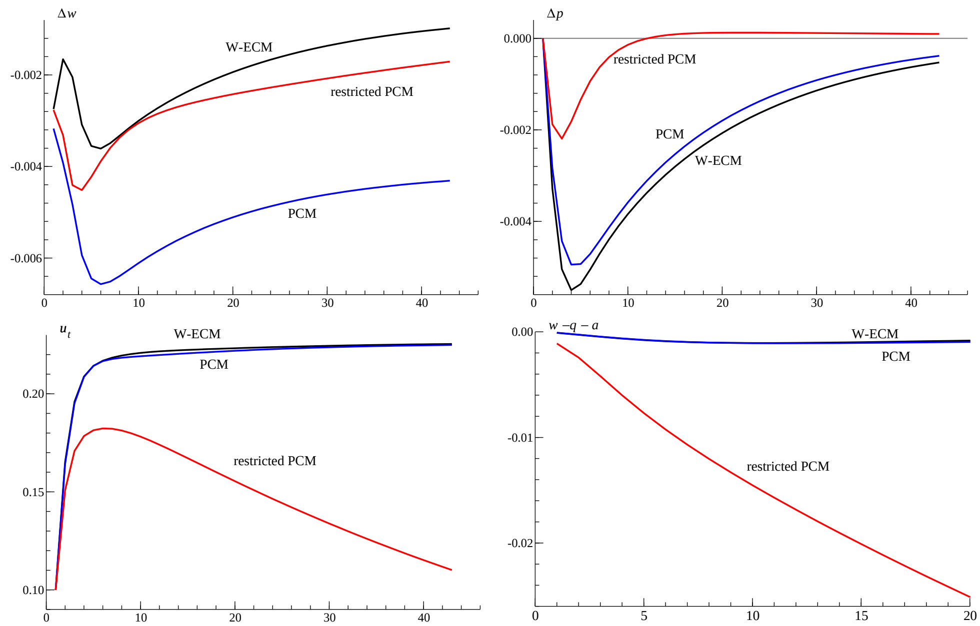

Figure 4: Dynamic multipliers of the econometric models PCM, W-ECM, and the restricted PCM, to a permanent exogenous 0.5 reduction in the unemployment percentage.

in this case, showing the force of extended equilibrium correction.

The graph for the restricted econometric PCM in panel c) shows the response pattern that corresponds to the accelerationist Phillips curve model and of the theoretical PCM steady state unemployment rate in equation (28). Since the steady state unemployment rate of this specification of the model only depends on the parameters of the wage Phillips curve (it is a "u-star") the shock to the unemployment rate has to be reversed completely before a new equilibrium can be restored. The single equilibrating mechanism of the model is the response of Δu_t to the lagged wage share, which therefore has to fall to a new steady state level. As can be seen in panels c) and d), the speed of adjustment is very low. For practical purposes it is as if the level of unemployment never returns to its initial and natural value. Thus, in the *restricted* PCM, corresponding to the standard natural rate Phillips curve model discussed above, the single equilibrating mechanism is extremely weak, making stationarity of u_t more of a formality than an important system property.

Bårdsen and Nymoen (2009b) show that the restrictions that separate the restricted PCM form the PCM were statistically rejected. Hence in this case, the natural rate hypothesis was rejected. It can be noted unlike other tests, which focuses on the slope of the wage or price Phillips curve, cf. e.g., Blanchard (2016), the system oriented test does not depend on non-constancy of the US Phillips curve to reach the conclusion. As such it is a new test, supplementing the earlier single equation tests.

b) Wage-price specification and optimal monetary policy

Akram and Nymoen (2009) showed how optimal monetary policy could be implemented in an econometric model of the Norwegian economy called Norwegian Aggregate Model (NAM), see Bårdsen and Nymoen (2009a). They also analyzed how much the predicted economic outcome depended on the specification of the supply side of the model, in particular as WP-ECM or PCM.

In the analysis, they used a theoretically derived rule for setting the interest rate i_t, cf. Akram (2010):

$$i_{t+h} = i_0 + \beta_{\varepsilon,H}\varepsilon_t + g_H(i_{t+h-1} - i_0),\ h = 0, 1, 2..., H. \tag{37}$$

This rule defines an interest rate path corresponding to a specific policy horizon H. The response coefficient

$$\beta_{\varepsilon,H} = (1 - g_H)\frac{\beta_\varepsilon}{(1 - \phi_\pi)},$$

determines by how much the interest rate must deviate initially from the neutral rate i_0 to counteract inflationary effects of a shock ε_t. A high value of the smoothing parameter g_H can be associated with a strategy of gradualism in interest rate setting. Thus, the parameters $\beta_{\varepsilon,H}$ and g_H depend on the policy horizon, H. The last parameter in the theoretical interest rate equation is ϕ_π, which represents the (objective) degree of persistence in the inflation shock.

In addition to the preferences about policy horizon and gradualism captured by (37), the optimal interest rate response is influenced by the user's choice of macroeconomic model, which can be thought of as determining β_ε. Akram and Nymoen (2009) makes use of three versions of an empirical macro model of the Norwegian economy, cf. Bårdsen and Nymoen (2009a). In one model version, nominal wage and price setting is modelled by the WP-ECM framework above. The two other version are: The model with wage and price Phillips curves, PCM in the figure, and a version with a vertical Phillips curve, not unlike the restricted PCM in the analysis of the US data.

Akram and Nymoen showed that econometric encompassing tests favour the WP-ECM model of the supply side, but also that the PCM and the restricted PCM appears to be well specified on their own terms, for example the residual properties of the two Phillips curve models do not signal any problems. Hence there is a question about whether the outcome of the encompassing test has any practical (or "economic"), significance, or whether this test of model adequacy only has an academic interest. The analysis suggest that the econometric test result contains valuable information.

By the use of model simulation, Akram and Nymoen identified a trade-off between price and output stabilization for different ranges of policy horizons. Specifically, in the case of WP-PCM and PCM the trade-off was in the range of 0 to 8 quarters. Policy horizons that are longer than 8 quarters appeared inefficient as both price and output stabilization could be improved by shortening the policy horizon. The opposite was the case for PCMr. In that case, the trade-off curve was associated with policy horizons that were longer than 6 quarters, while policy horizons shorter than 6 were inefficient.

Even though the efficiency frontiers for ICM and PCM were defined by almost the same policy horizon, the optimal horizon (for the given loss function) was found to be 3 quarters conditional on WP-ECM, but 6 quarters in the case of PCM. In the case of PCMr the policy horizon was eleven quarters.

Based on the above and other simulation experiments, Akram and Nymoen found that econometric differences bear heavily on model-based policy recommendations and was thus not merely of academic interest. Hence, monetary policy based on a misspecified model of wage formation may lead to substantial losses in terms of economic performance, even when policy is guided by gradualism, for example in the form of a long policy horizon.

c) Wage formation and business cycle "stylized facts"

Using empirical WP-ECMs for UK and and Norway, Bårdsen et al. (1998) analyzed the phenomenon of empirically unstable price/wage output correlations (r_{xy}) which had been noteds as a puzzle in business cycle analysis during the 1990s, cf. Blackburn and Ravn (1992), Englund et al. (1992), Andersen (1994, p 18-19) among others.

They demonstrated that the econometric models could resolve the puzzle of unstable r_{xy} values. A main conclusion was that only when shocks (impulses) come from the same part of the macro economic system (e.g., the demand side) can the pairwise correlations be expected to be stable for different samples, or to be consistent in cross-country comparison. Conversely, significant breaks in

r_{xy} values do not logically imply a structural change in wage and price setting.

Consequently, care must be taken in the usage of pairwise correlations in the calibration of theory driven models, for example real-business-cycle models and DSGE models. Indeed, as pointed out by Hendry (1995b), the practice of matching subsets of moments that are inherently non-constant induces a sample dependency in models that were intended to avoid exactly this difficulty.

d) Norwegian wage formation since industrialization

Nymoen (2017) modelled the Norwegian wage level over a long historical period, from 1900 to 2015. Norway was industrialized relatively late, and the data therefore covers most of its economic and institutional development since industrialization.

An empirical WP-ECM from the paper is shown in compact form in display (38). The endogenous variables are w, q, u and p. The exogenous variables are a, pi and four variables that capture *breaks* in wage and price setting and unemployment. In the same way as in the theory above, a, pi are modelled as random walks with drift (results not shown).

$$\begin{pmatrix} 1 & 0 & 0.03 & -0.5 \\ -0.36 & 1 & 0 & 0 \\ 0 & 0 & 1 & -0.98 \\ 0 & 0.33 & 0 & 1 \end{pmatrix} \begin{pmatrix} \Delta\widehat{w}_t \\ \Delta\widehat{q}_t \\ \Delta\widehat{u}_t \\ \Delta\widehat{p}_t \end{pmatrix} = \begin{pmatrix} 0.24 & 0 & 0.23 & 0.13 & 0 \\ -0.36 & 0.34 & 0.1 & 0 & 0 \\ 0 & -0.98 & 0 & 0 & 0.3 \\ 0 & 0.16 & 0.31 & 0 & 0 \end{pmatrix} \begin{pmatrix} \Delta a_t \\ \Delta pi_t \\ \Delta p_{t-1} \\ \Delta wc_{t-1} \\ \Delta u_{t-1} \end{pmatrix}$$

$$
+\begin{pmatrix} -0.08 & -0.01 & 0 & 0 & 0 \\ 0 & 0 & -0.14 & 0 & 0 \\ 0 & -0.37 & 0 & -0.27 & -0.24 \\ 0.04 & 0 & 0 & -0.02 & 0.03 \end{pmatrix} \begin{pmatrix} (wc-a-q)_{t-1} \\ u_{t-1} \\ q_{t-1} - 0.7(w-a)_{t-1} - 0.3pi_{t-1} \\ (w-p-a)_{t-1} \\ (pi-p)_{t-1} \end{pmatrix}
$$

$$
+\begin{pmatrix} 0.91 & 0 & 0 & 0 \\ 0 & 0.96 & & 0 \\ 0 & & 0.95 & \\ & & & 1.1 \end{pmatrix} \begin{pmatrix} wbreak_t \\ qbreak_t \\ ubreak_t \\ pbreak_t \end{pmatrix} \tag{38}
$$

The estimation method is Full Information Maximum Likelihood (FIML). To save space, the constant terms and the payroll-tax variable, which only enters the wage equation, have been omitted. For the same reason, standard errors of the coefficient estimates are not shown. But all individual 't-tests' would be significantly different from zero at the usual significance level.

The model (38) is an encompassing model, in the meaning that the restrictions it implies for the unrestricted VAR (reduced form) are jointly statistically insignificant. When the coefficient restrictions that define the PCM is imposed, the joint null of those restrictions are clearly rejected. However, as above, a relevant question is whether these restrictions are numerically significant'.

Figure 5 gives some indication of such numerical significance for the wage share. The solutions are obtained by dynamic solution of WP-ECM and the PCM versions of the model, with 1905 are the first solution period. The solutions appear to be almost identical for the first decades or so, but they gradually diverge, with the PCM-solution the least "I(0)"-like. This indicates that in this case, the extended equilibrium property is not very strong. Hence there is a significant difference in the solutions of the PCM and ECM, even before the hypothesis of vertical long-run Phillips curve is imposed, which would be a counterpart to the rPCM model in the model for US post-war wage and price inflation.

e) Forecasting

As noted by e.g., Nymoen (2019, Ch. 12), forecasting methods that often have been presented as alternative and competing, for example univariate time series models and larger systems of

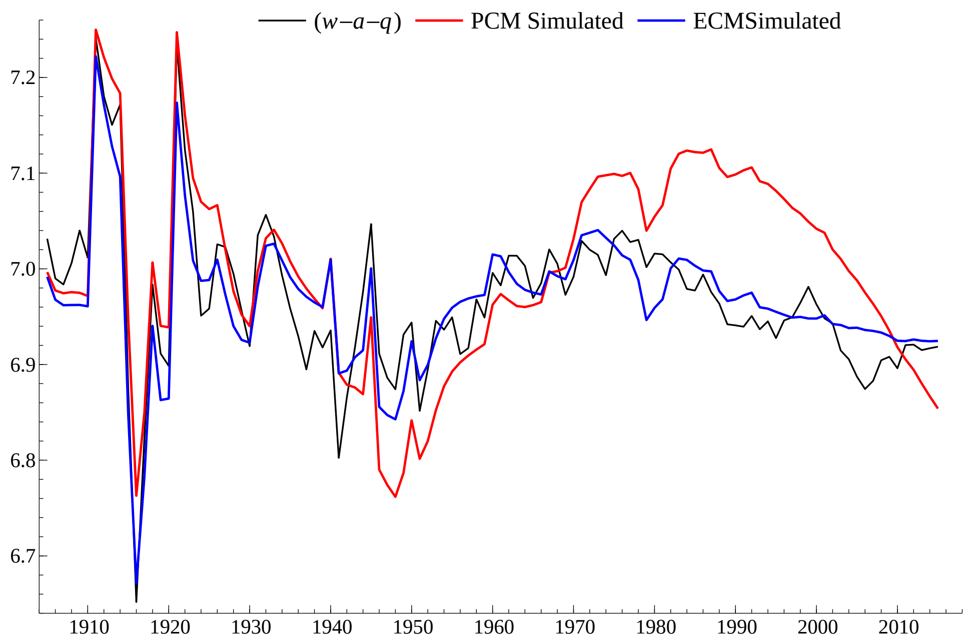

Figure 5: Dynamics simulation of the wage share

equations, in fact have much in common. For example, the forecasting function for a variable in a dynamic system, small or large, has a *glide path* interpretation. It's origin is an observed (or "nowcasted") starting point, and the end point of the glide path is the estimated long-run mean (*ie* mathematical expectation of the variable). The glide path of linear models is continuous between the starting point and the end point, but it is not monotonous in general. If the variable being forecasted is $I(1)$ non-stationary, only the starting point is well defined. However a cointegrating linear combination of $I(1)$-variables follows a well defined glide path when forecasted.

The forecasting properties of PCM and WP-ECM can be analyzed as special cases. Let the known initial values be given by the information set $\mathcal{I}_T^m$, where $m = PCM$ or $m = WP\text{-}ECM$ and assume that the model forecasts are conditional expectations based on $\mathcal{I}_T^m$. It then follows that for each endogenous variable, Y_i, all dynamic forecasts H periods ahead from T will be model dependent:

$$Y_{i,T+H|T}^{mf} = E(Y_{i,T+H} \mid \mathcal{I}_T^m). \tag{39}$$

As the forecast horizon H is extended towards infinity ("long term forecasts"), the forecasts glide toward the unconditional expectations, which will however still be model dependent.

To show the glide-path interpretation of the PCM and the WP-ECM forecasts, we can re-use the notation from the model typology section for the closed economy case (hence $\Delta p_t = \Delta q_t$), but adding constant terms (c_w and c_q):

$$\Delta w_t = c_w + \pi_w \Delta q_t + \tau_w \Delta a_t - \beta_w ws_{t-1} - \sigma_w u_t + \varepsilon_{wt} \tag{40}$$

$$\Delta q_t = c_q + \tau_p(\Delta w_t - \Delta a_t) + \beta_p ws_{t-1} + \varepsilon_{qt} \tag{41}$$

$$u_t = c_u + \varrho ws_{t-1} + \alpha u_{t-1} + \varepsilon_{ut} \tag{42}$$

For simplicity, we abstract from estimation uncertainty (imagine that we know the coefficients). We forecast the wage share and the unemployment rate H periods ahead from T: ws_{T+h} and u_{T+h} $h = 1, 2, ..., H$. To simplify notation, let $\alpha = \tau_p = \tau_w = 0$ let g_a denote growth rate of productivity as before. This gives, for the wage share forecasts:

$$ws_{T+H}^{\text{WP-ECMf}} \underset{H \longrightarrow \infty}{\longrightarrow} E^{\text{WP-ECM}}[ws_t] = \frac{(c_w + (\pi_w - 1)c_q - \sigma_w c_u - g_a)}{-\{(\pi_w - 1)\beta_p - \beta_w - \sigma_w \varrho\}}, \tag{43}$$

$$ws_{T+H}^{\text{PCMf}} \underset{H \longrightarrow \infty}{\longrightarrow} E^{\text{PCM}}[ws_t] = \frac{(c_w + (\pi_w - 1)c_q - \sigma_w c_u - g_a)}{\sigma_w \varrho}, \tag{44}$$

Clearly, the two end-points points of the forecast are not identical. Since the unemployment forecasts obey the same equation (for the two models):

$$u_{T+H}^{mf} = c_u + \varrho ws_{T+H}^{mf} \tag{45}$$

the end-points of the unemployment forecasts will also be different: $u_{T+H}^{\text{WP-ECMf}} \neq u_{T+H}^{\text{PCMf}}$. For the PCM specifically, we get:

$$u_{T+H}^{\text{PCMf}} \underset{H \longrightarrow \infty}{\longrightarrow} \frac{(c_w + (\pi_w - 1)c_q - g_a)}{\sigma_w} \tag{46}$$

i.e., the NAIRU rate above. In the case of $\pi_w = 1$:

$$u_{T+H}^{\text{PCMf}} \underset{H \longrightarrow \infty}{\longrightarrow} \frac{(c_w - g_a)}{\sigma_w}, \text{ natural rate} u^*. \tag{47}$$

It follows that in a stationary world with no structural breaks in the forecast period, only one set of model based forecast will be unbiased. It will be the forecasts of the model that best approximates the DGP. The forecasts of the other model of the supply side will have larger mean squared forecasted error (MSFE).

A main source of systematic forecast errors in macro models are structural breaks in the DGP that happen in the forecast period or close to (or in) period T. It is impossible to protect forecasts against the first type of break. The second type can be difficult to discover and assess in practice. In this situation it is not certain that the model which best approximates the DGP within sample also has the lowest MSFE. The literature has shown that models which only use differenced macro variables, so called dVARs, may be more robust than the forecasts of a model that makes use levels variables, i.e., in the way that the WP-ECM does, see e.g., Clements and Hendry (1999).

An attempt to analyse the empirical forecast performance of WP-ECM and PCM was reported in (Bårdsen et al., 2005, Ch. 11). The analysis showed that although the PCM shared some of the robustness of dVARs, it also embodies equilibrium correction in the form of natural rate dynamics. Since that form of correction mechanism was rejected empirically, the PCM forecasts were harmed both by excessive uncertainty (from its dVAR aspect), and by their econometric mis-specification of the equilibrium-correction mechanism in wage formation.

5 The New Keynesian Phillips Curve (NKPCM)

A common feature of the WP-ECM and the PCM is that wage changes (wage inflation) are jointly determined with changes in the price level (inflation). The joint dependency is not necessarily simultaneous, but the models capture that one cannot have wage inflation without price inflation, and vice versa. Looking back at the typology (section 3), this is not the case for the New Keynesian Phillips curve model, (10)-(11). In that model, inflation is not explained by wage changes. The wage level (relative to the value of labour productivity) does however play a role, since the wage share is the forcing variable in the NKPCM inflation equation. Instead of an equation for nominal wage change, the NKPCM is specified with a first order autoregressive process for the wage share (ws_t):

$$ws_t = (1 - \beta_{w3})ws_{t-1}$$

Hence, taken at face value the NKPCM is tacit about how the wage share and the nominal wage hang together with inflation and the other variables in the macro economic system. However there is nothing hindering a symmetric argument, leading to a wage version of the New Keynesian Phillips curve, Galí (2011). The combination of the two NKPCMs defines a model of the supply side which it is possible to compare with the WP-ECM and PCM.

a) The New Keynesian Phillips curve

The best known version of the NKPC is due to Galí and Gertler (1999) and is defined as:

$$\Delta p_t = c_p + \underset{>0}{\pi_p^f} E_t[\Delta p_{t+1}] + \underset{\geq 0}{\pi_p^b} \Delta p_{t-1} + \underset{>0}{\beta_p} ws_t + \epsilon_{p,t}, \tag{48}$$

where we have used the notation of the typology, augmented by an intercept and a random error-term added.[10] A hallmark of this theory is that the forward term Δp_{t+1}^e in the typology is defined as $E_t[\Delta p_{t+1}]$, i.e., the expected rate of inflation in period $t+1$, given the information available for forecasting at the end of period t.

In Galí and Gertler (1999), and several others, there is no error-term in (48) (implying $\epsilon_{p,t} = 0$). We follow custom and refer to it as the exact form of the structural equation. The version with the error-term $\epsilon_{p,t} = 0$ specified as e.g., a gaussian white noise process conditional on time t information, is referred to a the inexact form of the NKPC. The distinction between exact and inexact form becomes important for some methods that test the restrictions that the rational expectations solution implies for a cointegrated VAR, see Boug et al. (2017) and the references therein.

Setting $\epsilon_{p,t} = 0$ is probably not meant as a claim about having the exact theory. However, is does reflect a line of thinking saying that the empirical verification of the properties of an random error-term in the structural equation is unimportant, for example because robust estimation methods is in any case used to estimate the coefficients, e.g., by GMM. However there is no logic saying that implicit, imprecise or "weak" probabilistic assumptions render the conclusions of en empirical study (quantification) less vulnerable to statistical mis-specification. When we speak about statistical induction, the premium is on testability of assumptions. When validated, stronger assumptions may give a much more effective way of learning from the data than weaker assumptions do, Spanos (2020, § 5.3).

In the approach we use below the "inexact form" of the structural inflation equation is used

[10] We have dropped the "model 3" part of the notation, since it is clear that we are looking at the NKPC in this section.

without complicating the derivation, estimation or empirical evaluation. The econometric inflation equation we obtain is accompanied by the specification of the process for the forcing variable ws_t as a second order autoregressive process, thus generalizing the notation in the typology in a way that conforms with many applications:[11]

$$ws_t = c_{s0} + c_{s1} ws_{t-1} + c_{s2} ws_{t-2} + \varepsilon_{s,t}, \tag{49}$$

The solution for Δp_t, using (48) and (49) becomes:

$$\Delta p_t = b_{p0} + b_{p1} \Delta p_{t-1} + b_{p2} ws_t + b_{p3} ws_{t-1} + \varepsilon_{p,t}. \tag{50}$$

This equation is an autoregressive distributed lag model (ADL) model equation, well known from dynamic econometrics. Given the assumptions of the theory, ws_t is uncorrelated with the error-term $\varepsilon_{p,t}$ which is proportional to the error-term in (48).[12] Hence, if it is assumed that the structural error-term $\epsilon_{p,t}$ is Gaussian white-noise, $\varepsilon_{p,t}$ in (50) is also normally distributed. Hence, OLS estimation of (50) gives approximate maximum likelihood estimators which are consistent. Formal (by encompassing tests) or informal comparison with competing specifications is therefore straight forward to do., cf. Nymoen (2019, Ch. 7.11).

In the literature that has assessed of the New Keynesian thoery of inflation, this route was largely been overlooked. Instead researchers have focused on IV and GMM estimation of the structural equation (48), see Bårdsen et al. (2004), Castle et al. (2013) and Mavroeidis et al. (2014) which overviews the results for the π^f coefficient in particular. Nevertheless, as we shall see below, when Galí (2011) introduced the New Keynesian *wage* Phillips curve, he proposed to estimate a solution equation similar to (50), as a way of obtaining empirical validation of his wage model.

Returning to the solution equation (50), it is worth noting that the coefficients of (50) are combinations of the parameters of (48) and (49) see e.g., Bårdsen et al. (2005, Appendix A) building on Pesaran (1987) and Nymoen et al. (2012). b_{p1} is identical to the stable root (r_1) of the associated characteristic equation of (48). The two distributed lag coefficients depend on the parameters of both (48) and (49).

[11]In the first order case, $c_{w1} = 1 - \beta_{w3}$ in the notation of the model typology

[12]$\varepsilon_{p,t} = 1/(\pi^f r_2)\epsilon_{p,t}$, the scaling factor is $1/(\pi^f r_2)$ where $r_2 > 1$.

One final point is that the constant term b_{p0} in (50) is also a confluent term. Setting $c_p = 0$ without loss of generality, it can be shown that (cf. Nymoen (2014)):

$$b_{p0} = \frac{\beta_p}{\pi_p^f}(\frac{1}{r_2 - 1})\mu_s, \tag{51}$$

where $r_2 > 1$ is the so called unstable root and μ_{ws} is the expectation of the wage-share ws_t, i.e.:

$$\mu_s = \frac{c_{s0}}{1 - c_{s1} - c_{s2})} \tag{52}$$

Hence, if there is a change in the mean of the process of the forcing variable, there should theoretically also be a change in the constant term of the solution equation of inflation, (50). Since structural breaks in the form of location shifts are common in economic time series, this represent an additional implication of the theory which is testable.

b) The New Keynesian Wage Phillips Curve

The wage-NKPC, due to Galí (2011) (with error-term added) is:

$$\Delta w_t = c_w + \underset{>0}{\pi_w^f} E_t[\Delta w_{t+1}] + \underset{\geq 0}{\pi_w^b} \Delta\bar{\pi}_{t-1} - \underset{>0}{\sigma_w}\ u_t + \epsilon_{w,t}, \tag{53}$$

where $\Delta\bar{\pi}_t$ is "the price inflation variable to which wages are indexed",Galí (2011, p 441).[13]

The forcing variable in (53) is the unemployment rate, with generating equation:

$$u_t = c_{u0} + c_{u1}u_{t-1} + c_{u2}u_{t-2} + \varepsilon_{u,t}, \tag{54}$$

where $\varepsilon_{u,t}$ can be assumed to be Gaussian white-noise without loss of generality. The solution for Δw_t, given (53) and (54) is:

$$\Delta w_t = b_{w0} + b_{w1}\Delta\bar{\pi}_{t-1} + b_{w2}u_t + b_{w3}u_{t-1} + \varepsilon_{w,t}. \tag{55}$$

In the same way as for (50), the coefficients of (55) depend on the parameters of (53) and (54). In particular the constant term b_{w0} contains the expectation of u_t. Hence, if there is a structural break

[13]To save notation, the mean of u_t is subsumed in c_w. Equation (14) in the paper also includes the term $E_t[\bar{\pi}_t] = \bar{\pi}_t$, which is however omitted from the solution expression (18) in the paper, corresponding to (55).

in for example c_{u0}, this theory predicts that also b_{w0} should break.

However, the point made by Galí (2011) was more in the direction of seing (55) as a reduced form wage inflation that rested on better theory than given hitherto for the typical US wage Phillips curve.[14]

A possible objection to the suggestion to use (55) for econometric evaluation of New Keynesian theory is that is based on the exogenous AR(2) process (54). After all, basic macroeconomic theory implies that wage inflation can be expected to be Granger-cause unemployment, and the DSGE model of which wage NKPCM is meant to be part of, is no different in this regard. Hence it may seem self-contradictory to assume an exogenous unemployment process as a step in the derivation of the solution (55)). One way around this is Campbell and Schiller's method for assessment of present value relations, which however requires the (incredible) assumption that (53) holds exactly, cf. Galí (2011).

However, it is standard in the New Keynesian DSGE model literature to assume that the data generating process (DGP) belongs to the unique stable solution of the multiple-equation model that allow for mutual feedback. Under this assumption, the DGP takes the form of a globally asymptotically stationary VAR, see Bårdsen and Fanelli (2015). There are implied *final equations* for every endogenous variable of the VAR. The final equations are difference equations that have identical homogenous parts, while the non-homogenous parts are (by definition) heterogenous between variables, see e.g., Nymoen (2019, p. 142), Hendry (1995b, p. 340).

In this light, equation (54) can be interpreted as the final form equation of u_t, from a stationary VAR with first order dynamics with white-noise errors. Interestingly, the VAR assumption is also made by Galí (2011) in order to construct a time series for so called fundamental wage inflation by using Campbell and Schiller's procedure.

One caveat is that a VAR with white-noise errors have final equations which has with moving-average errors, i.e., the non-homogenous parts are composites of the VAR errors. Hence, strictly speaking when we interpret (54) as a final form equation, $\varepsilon_{u,t}$ is a moving-average process. Since that property of (54) was not taken into account in the derivation of the solution equation (53), the result mentioned above about $\varepsilon_{w,t}$ being a white-noise error term may need to be modified.

[14] Galí allows for an autocorrelated error-term in his estimated equation, due to "measurement error in the wage inflation data" (p.452) and not to potential mis-specification. However, Table 1, only reports OLS results and there are no diagnostic tests of residual mis-specification.

However, it has been shown that a moderately over-parameterized AR-model gives a close approximation of an ARMA model, i.e., in terms of fit and with respect to the linear properties of the time series, see Wahlberg (1989), Hendry (1995a, Ch 15.7) and the references therein. Consequently, proceeding from the assumption that (54), with $\varepsilon_{u,t}$ as white-noise, is defensible as an approximation to a first order ARMA model of u_t. The cost of the approximation is perhaps some loss of parsimony.

Hence, a practical method is to include all significant lags in the final form equation of u_t, subject to the residuals being empirically white-noise. The lag order can then be set to for example three, instead of two as in (54). The extended dynamics in the final form equation creates no problems for the derivation of the solution equation for Δw_t. The consequence will be that one or more additional lags in u_t is included in the equation. Hence, if the final form equation is of order three, there will be a distributed lag of order two in u_t in the wage change equation, instead of the first order distributed lag in (55)).

A similar remark can be made for the price New Keynesian price Phillips curve (50), which can be augmented with longer lags in the ws_t if that helps in specification of a non-misspecified model equation.

c) Examples of empirical New Keynesian solution equations

Collecting equations we can formulate a "NKPCM-system":

$$\Delta p_t = b_{p0} + b_{p1}\Delta p_{t-1} + b_{p2} ws_t + b_{p3} ws_{t-1} + \varepsilon_{p,t}, \tag{56}$$

$$\Delta w_t = b_{w0} + b_{w1}\Delta \bar{\pi}_{t-1} + b_{w3} u_t + b_{w3} u_{t-1} + \varepsilon_{w,t}, \tag{57}$$

$$ws_t = \Delta w_t - \Delta p_t - \Delta a_t + ws_{t-1} \tag{58}$$

$$\bar{\pi}_t = 0.25\Delta_4 p_t \tag{59}$$

where the two last equations are identities that help close this New Keynesian model of the supply side. In (59) the wage-indexation term $\bar{\pi}_t$ is specified as the year-on-year price inflation, which was used by Galí (2011). [15]

From the perspective of our paper, a main point is that the equations of the the NKPC system can evaluated as empirical models, by the use of standard methods of time series econometrics.

[15] cf. Galí (2011, Table 1)

As noted, based on the underlying theory, the two error-terms are white-noise and homoskedastic. Nevertheless Galí (2011) use robust standard error in tables with estimation results, with reference to measurement errors in the wage data. However, there are several other sources of mis-specification, and Spanos (2018) reminds us, there is very little robustness without statistical adequacy.

As an illustration, we report an estimated model (56) using the data and sample period (1960(1)-1997(4)) in Galí and Gertler (1999):

$$\Delta p_t = \underset{(0.000447)}{0.00122} + \underset{(0.0398)}{0.8732} \;\; \Delta p_{t-1} + \underset{(0.028)}{0.06879} \;\; \Delta ws_t + \underset{(0.0154)}{0.0227} \;\; ws_{t-1} \tag{60}$$

$$\hat{\sigma}100 = 0.27 \quad R^2 = 0.812 \quad T = 152$$

Given the theoretical pre-eminence of the wage share as the forcing variable in the NKPCM, a puzzling result in this equation is the statistical insignificance of coefficient of ws_{t-1} (i.e., $(b_{p2} + b_{p3})$): t-value 1.5. The relative weakness of the wage-share as a forcing variables has been noted in the literature that has investigated the NPCM by estimating the structural form equation, see Bårdsen et al. (2004) and Castle et al. (2013) among others. In practice, the fit of the model equation is well approximated by an AR(1) process, meaning that there is little "inherited" persistence from the expectation formation about forcing variables, and mainly "intrinsic" inflation persistence, Fuhrer (2006).

Standard mis-specification test for residual autocorrelation and heteroskedasticity are significant when they are applied to (60).[16]. As we have seen, the underlying New Keynesian theory allows inclusion of more lags in the forcing variable, if that improves the model specification. However, adding three lags draws a blank since the test of joint significance becomes: $F(3, 145) = 1.3398[0.2638]$.

Finally, we can investigate the implication that structural breaks in wage-share process should induce breaks in the solution equation of Δp_t. A feasible approach is to use the impulse indicator saturation (IIS) method which is a part of the automatic variable selection algorithm Autometrics, Doornik (2009), Hendry and Doornik (2014). Estimating the final form equation (49) of ws_t using IIS with overall significance level 1 % Autometrics returns 11 indicator variables. When this set of indicators for structural breaks in the mean of inflation was added to (60) only two dummies

[16]Applying robust standard error reduces the implied t-value of the lagged wage share a little more, to 1.3

had significant t-values: the dummies for 1977(4) and for 1978(2). Conversely, application of ISS-Autometrics to (60) gave 12 indicator variables, but only one of them, for 1977(4), was also a break in the equation of the forcing variable ws.

The probability of finding spurious breaks can be controlled by choosing a lower overall significance level in the algorithm. Repeating the above estimation with significance level 0.1 %, Autometrics finds no breaks in the ws equation, but returns three break dummies for (60): 1972(2), 1974(3) and 1977(4). In summary there is less "syncronization" between the breaks in the Δp_t equation and in the process of the forcing variable than the New Keynesian theory predicts.

The estimated version of the wage NKPCM (57) shown in (61) fares better in terms of the strength of the forcing variable. The sum of the two unemployment coefficients is statistically significant, and is quite sizeable. Of course, this is not surprising considering the empirical success of the wage PCM when estimated on US data.

$$\Delta w_t = \underset{(0.0016)}{0.0128} + \underset{(0.07435}{0.945149} \ \Delta \bar{p}_{t-1} - \underset{(0.1234)}{0.179521} \ \Delta u_t - \underset{(0.02779)}{0.142321} \ u_{t-1} \tag{61}$$

$$\hat{\sigma}100 = 0.467 \quad R^2 = 0.549 \quad T = 152$$

Using a 1 % significance level, Autometrics-IIS finds 8 break indicators in the final form equation of u_t, however only one of them, 1960(1), is statistically significant when added to the New Keynesian wage Phillips curve (61). In the same way as for the price NKPC, this indicated less pass-through than theory predicts, of breaks in the mean of the forcing variable to the wage equation. This does not however imply that the price NKPC has constant mean. With the 1 % level of overall significance, Autometrics-IIS finds 8 break dummies in this equation as well, but as we have seen only one of them, (1960(1)) is found to be a structural break also in the wage equation.

d) Tests based on the structural equation and encompassing tests

The price and wage versions of the New Keynesian Phillips curve are examples of economic models that include expected future values to explain current outcomes. As noted above, models of this type are estimated by IV or GMM after replacing the expected value by the actual future outcome. The focus of this literature has been on the empirical evidence on inflations expectations in the (price) New Keynesian Phillips curve. Mavroeidis et al. (2014) is a comprehensive survey.

One feasible way of testing the structural equation more broadly is by running encompassing tests against other relevant models of wage and price setting. Specifically, the following procedure can be followed:

1. Assume that there exists a set of variables $z = [z_1, z_2]$ where the sub-set z_1 is sufficient for identification of the structural NKPCM model equation.

2. Using the identifying sub-set z_1 , estimate by IV (or GMM) an augmented NKPCM where the augmentation consists of the other sub-set of variables, z_2.

3. Under the hypothesis that the NKPCM is the correct model, the coefficient vector of z_2 in the augmented NKPCM equation is zero, and the numerical and statistical significance of the lead-variables should be more os less be that same as found in 1.

In addition to inspection of the t-values of the relevant variables, one can also estimate the over-identified NKPC (all z-variables are used as instruments), and interpret the Sargan-specification test, Sargan (1958,1964), Davidson and MacKinnon (2004, 8.6), or the equivalent J-test Hansen (1982), as encompassing tests. These test statistics should be insignificant if the NKPCM is the correct model equation.

In terms of practical implementation one could take advantage of existing results on wage and price modelling using cointegration analysis which readily delivers z_2-variables in the form of linear combinations of levels variables. In other words, they represent "unused" identifying instruments that go beyond information sets used in the estimation of structural NKPCM. An example is given in Bårdsen et al. (2005, Ch. 7.5), who used instrumental variables from the study by Bårdsen et al. (1998) (cited above) to test the encompassing properties of an empirical NKPC model equation for the UK economy, specified by Batini et al. (2005).

Of course, for encompassing tests to affect beliefs about wage and price setting, there must be some common ground, in the form of mutual recognition of relevance of empirical findings, and of the results of pre-existing studies. It seems that sometimes the pre-eminence of theory in the specification of empirical econometric models has intervened in a way that have reduced the role of encompassing testing in the progress of the discipline.

An alternative approach, which does not require variables from other studies to be introduced, but which also uses the equations with explicit lead-terms, utilize the implicit parameter invariance assumption the NKPCM model. The point is that since there is no allowance for unanticipated regimes shifts in the reduced forms of the lead-variables in either (48) or (53), a significant and sizable IV estimated coefficient of Δp_{t+1} or Δw_{t+1} can potentially be spurious. Castle et al. (2013) proposed using a 2-stage procedure. The first stage applies IIS-Autometrics to the reduced form of Δp_{t+1} to detect the presence of any unmodelled outliers or location shifts; and the second is to test of their presence in the structural equation (48). When the method was applied to both U.S and Euro-area price NKPCs, Galí and Gertler (1999) and Gali et al. (2001) the previous results became radically altered.

The method that was introduced in paragraph c) above is complementary to these tests. As noted, that new test does not require identifying assumptions for estimation, or over-identifying instruments for testing. It utilizes the implication if the theory model is correct, a break in the mean of the forcing variable should go together with a break in the constant term of the solution equation. As structural breaks are known to be frequent in macro economics, the test should have some force.

6 Concluding remarks

This paper offers a short and incomplete history of wage formation models summarized in three main conceptualizations: Phillips Curve Models (PCMs), wage-price equilibrium correction models (WP-ECMs) and New Keynesian Phillips curve models (NKPCMs). They make out a typology of candidate models of the supply side of medium term macroeconometric models. The papers continued by presenting the salient features of PCMs and WP-ECMs. Some of these features are shared between the models (e.g. dynamic joint causation), while other properties and implications are markedly different, e.g., the definition and role of the steady-state rates of wage inflation and of unemployment. PCMs and WP-ECMs remain to be powerful conceptualizations of some of the main debates about how national macro economic systems function, and are therefore associated with competing beliefs about which policy strategies and measures are effective. Ultimately also about which targets of economic policies that are feasible to pursue.

The common statistical model used for presentation of PCMs and WP-ECMs are the integrated VAR with a causal solution (i.e., only known initial conditions required). The difference between the PCM and WP-ECM can to some extent be brought down to specified assumptions about unit-roots, co-integration and exogenity (which variables take the main roles in the adjustment of disequilibrium).

NKPCMs at first sight cannot be placed in the framework of a VAR with causal solution. This is due to its main definitional characteristic, the lead-term in inflation (wage change in the case of the wage NKPCM). However, when the lead-variable is modelled by the hypothesis of rational expectations, its solution can be assumed to have stationary and causal VAR representations. In this paper, we follow that approach to show that the implied solution-equations of the hybrid price and wage NKPC take the form of "ordinary" dynamic regression models for wages and price, that can be compared (tested against) WP-ECM and PCM models.

The econometric treatment of wage formation for inclusion in macroeconometric models began during an era of full employment and relative stable prices in the western post war economies. The policy issue was then wether near full employment could be maintained without creeping inflation evolving into wage-price spirals. As inflation became recognized as endemic, the focus shifted in the direction of analysing the role of wage formation and labour market institutions in the determination of the equilibrium unemployment rate, understood as the rate of unemployment that was reconcilable with stable inflation. More recently, but already two decades ago, the "inflation problem" quite suddenly disappeared in many modern economies. That change may have had less to with inflation targeting central bank policies than many believe, and more with China taking over the role as the workshop of the world and other changes towards globalization going bach even longer in time.

During these epochs and periods of transition, the supply side of macro econometrics models have evolved and adapted. The changes can be interpreted as answers to the needs expressed by models users. However, there are also examples of direct influence from academic economics. The rise of the NKPCM, as part of the wider DSGE model movement, perhaps stands out as the main example of a theoretically driven change. However, earlier in the history of wage modelling, WP-ECM models were strongly influenced by the spread of the bargaining theory in economics, and the coming together of dynamic econometrics modelling and VARs (through cointegration methods).

In the present lull, where it has become normal to expect below-target inflation and where low interest rates are expected to last "forever", it may be easy to lose sight of the latent and complex dynamics in wage and price setting. However, if not exactly on the horizon, inflation could be on the way back, Goodhart and Pradhan (2020). In that case, although a future inflationary epoch will be different from the historical ones, the advances already gained in empirical econometric modelling in this field will be a solid basis for the necessary revisions of knowledge, skills and understanding overall.

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