The Exchange rate in a small open commodity exporting economy*

Thor Andreas Aursland, Roger Hammersland and Dag Kolsrud

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Abstract

This paper has several objectives. First, it aims at investigating whether and to what extent it is possible to find support for a New-Keynesian rational expectation model by resorting to a purely frequentist estimation approach, not imposing a priori restrictions on the model parameters. Second, and given the fact that a model estimated this way turns out to be inconvenient for a direct operationalization of the rational expectation paradigm in a wider macro econometric model environment due to issues with instability, it looks at how this problem can be overcome by either assuming an adaptive expectation formation scheme or resorting to a framework where the solution of a saddle path stable general equilibrium model is embedded as a separate block in a more comprehensive macro econometric model environment.

As far as the results of resorting to a frequentist estimation method to estimate empirical models informed by rational expectation general equilibrium models are concerned, our analysis suggests that it in specific cases may be possible to freely estimate a structural model that comes close to being consistent with several features of a New-Keynesian rational expectation model. In particular, in the framework of a general equilibrium model of a two-country two-markets economy we do find - or at least we cannot reject - support for the existence of a Phillips curve. This is a finding that questions whether the conclusions of some former research can be due to a simultaneity bias in design and estimation. As far as the operationalization of forward looking expectations in macroeconomic models is concerned, our analysis indicates that one may circumvent the inherent problem of stability by both utilising an adaptive expectation formation scheme and incorporating the saddle path solution of a rational expectation general equilibrium model directly as a sub model within the wider macro econometric environment. However, such a strategy does seem to come at a cost, as there is no way to preserve a two-way communication between the general equilibrium block and the encompassing macro model without compromising the overall model's stability.

1 Introduction

Much recent econometric work concerning exchange rate modelling revolves around models based on uncovered interest parity or rather deviations from it, the latter often in terms of the existence of a foreign exchange risk premium. Limiting our reach to empirical models constructed utilizing classical estimation methods, not imposing a priori restrictions (distributional or otherwise) on the model parameters, these are often constructed based on one-equation-at-the-time

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design processes, or processes not taking properly into account the fact that the exchange rate is the outcome of a simultaneous causal interaction process, involving a set of interdependent endogenous variables. Nominal and real exchange rates, inflation and real interest rates are after all endogenous macroeconomic variables; their dynamic path, and the determination of expected future inflation and real interest rates, all depending on the simultaneous structure of an underlying hypothetical structural data generating process (DGP). As a corollary of this, economic agents when forming their expectations of future endogenous variables, should take such a contingency into account, based upon an appropriate economic perception - or understanding - of how this imaginary structure might look like. Likewise, when designing and estimating models for explaining exchange rate fluctuations, model builders should base their efforts on a theoretical understanding suitable of capturing such a reciprocal interaction scheme.

To satisfy such a requirement this paper aims at designing and estimating a model for the krone/euro exchange rate that in principle is based on a fully structural and simultaneous understanding of the underlying mechanism driving the joint distribution of exchange rates, prices and interest rates. To be more precise in this respect, this means that the model developed herein is based on - or rather informed by - a New-Keynesian model set-up of a two-country two-markets economy where the exchange rate is determined jointly with prices and interest rates based on a rational expectation general equilibrium framework. Within such a framework representative consumers and monopolistic producers are supposed to have equal and symmetric preferences and technology. They are also assumed to exert optimizing behaviour, in the sense of respectively maximizing their utility and the discounted present value of their expected flow of future profits.

However, unlike the literature where these types of models are brought to data without letting data play a significant role - that is, beyond what is allowed for by Bayesian estimation of some of the models' structural parameters - this paper advocates a procedure that renders possible a full-fledged frequentist approach to the task of confronting structural models with data, not imposing a priori restrictions (distributional or otherwise) on the structural model parameters prior to estimation. This has been done not only to test the suitability of frequentist-driven methods for estimating customized versions of structural general equilibrium models, but also to later implement the sub-model thus created in a large macro econometric model of the Norwegian economy called KVARTS (2). In order to achieve this, theory is assumed to apply in the long run and to define an attractor that is embedded in a more general dynamic setting than indicated by the theoretical dynamic equilibrium model *per se*. To be more explicit this means that we have embedded the deviation of the vector of endogenous variables from its theoretical long-run structural solution as an attractor in a structural vector error correction model (SVECM) where all the parameters are estimated freely.[1]

By following such an approach, it is directly surprising to us at least how close we are to being able to freely estimate a structural model that is close to being consistent with some features of a New-Keynesian rational expectation model framework of a two-country two-markets economy, that is without resorting to either calibration or restricting the parameters to lie within a narrow distribution around a prior understanding of their signs and quantitative size. Admittedly, the dynamics deviate significantly from the one that follows from the New-Keynesian optimization problem per se and it has been necessary to equip the model with a number of deterministic variables such as dummies for structural breaks and seasonality to cope with the worse cases of unexplained systemic information. It should also be mentioned that the estimated

[1] One should note that this approach deviates from the one advocated in (6) which aims at preserving the model's dynamic and systemic properties while being flexible in the design of the models' long-run structure. It deviates also from (5) in that it seeks to model the simultaneous structure based on a purely frequentist SVECM approach without having a rigorous evaluation of an underlying DSGE model in mind. Compared to (5) one may therefore say that we in this paper have swaped the hypotheses, the null in our paper being the SVECM model while the alternative amounts to a strict DSGE interpretation of the results.

persistence in the exchange rate equation of our model is not consistent with that which follows from theory, in the sense that it implies a stable backward-looking exchange rate process and not one that would guarantees a unique stable saddle path solution in a three dimensional structure with two free variables . Taken at face value, one would perhaps have though that this should imply the existence of multiple equilibria and self-fulfilling prophecies, but simulations of the model based on a Gauss-Seidel iterative scheme across all the observations of the sample and imposing the same kind of parametrical restriction on a fully consistent version of the theoretical rational expectation model, is suggestive of explosive roots and instability.

However, when this is said, the freely estimated system implies the existence of a Phillips curve that is fully consistent with a New-Keynesian rational expectation equilibrium framework, a circumstance that forms the basis for the importance of forward-looking expectations in the exchange rate, price and interest rate formation *per se*. The estimated policy rule is moreover consistent with an extended Taylor rule with interest-rate smoothing ((9)) where, in addition to the Taylor principle being fulfilled, the interest rate is a function of a proxy for the output gap and the divergence of actual inflation rates from a target inflation rate. As far as the exchange rate equation is concerned it states that the real exchange rate in the long run is a function of the real interest rate and a risk premium, the first statement being in accordance with a reasonable and testable restriction on the structural parameters of the underlying theoretical model and the second alluding to a premium that is captured by the oil price in the short run and by the value of oil- and gas exports as a proportion of the value of total exports in the long run.

As far as the implementation of the above mentioned structure in KVARTS is concerned this has turned out to be difficult without either forcing the solution to be in accordance with the saddle path of an underlying rational expectation general equilibrium model being fully consistent with our theoretical model set up, or resorting to an adaptive expectation formation scheme that involves the use of backward looking AR- or VAR -models, the last alternative overruling the role of the Phillips curve as an expectation generating relation. Accordingly, the implementation of forward-looking behaviour in KVARTS has been undertaken by resorting to two different and reciprocally excluding approaches. One by linking our estimated empirical model to an adaptive expectation formation scheme using AR-models or VARs. And one where we implement the saddle path of a calibrated version of the New-Keynesian general equilibrium model that forms the basis of our empirical analysis. Simulations using KVARTS suggest that a SVECM model with forward-looking expectations and where expectations are adaptive in the sense that inflation expectations are formed in accordance with an estimated backward looking autoregressive process for inflation, gives impulse responses that deviate to a small degree from a model without a Phillips curve and where expectations are not modeled explicitly. When implementing forward-looking expectations in a model that is fully consistent with a New Keynesian rational expectation model, responses to shocks will depend on the degree of price rigidity. At low price rigidity, much of the adjustment to the shock will come via inflation and be quick, while the adjustment will be more gradual and to a greater extent come via the exchange rate at high price rigidity.

In the following, we explain in Section 2 the theoretical framework used throughout as an overall framework for the analyzes undertaken in this paper. In Section 3, we discuss data used in design and estimation of the empirical SVECM model. Section 4 documents the results of the SVECM modeling and its implementation in KVARTS as well as the results of building in a saddle path stable general equilibrium model for the simultaneous determination of the euro/krone exchange rate, relative prices and interest rate in KVARTS. Section 5 summarizes and concludes.

2 Theory

As a theoretical back drop for the modelling of the Norwegian Krone/Euro exchange rate we have chosen to use a two-country two-markets New-Keynesian model set up.[2] The aggregate relationships in this model are derived from the underlying decisions of households and firms, where each country produces a range of products all being produced by a country specific monopolist. Furthermore, in this model labor markets are competitive and workers are mobile between sectors within the economy. The model is derived from optimizing behaviour of infinitely lived representative households and monopolistic firms under uncertainty and with rational expectations. The model is further based on the assumption that not all firms are able to adjust nominal prices every period. The firms cannot therefore be perfectly competitive price takers, as they will need to have power to set their own prices for such a contingency being the case. Because firms have power to set their own prices they will set them above average and marginal costs - and earn a profit - each time they are allowed to change them. When firms subsequently do not get the opportunity to change their prices, they will be willing to sell the amount demanded ex post. That is, as long as demand does not exceed the point where marginal costs exceed the ex ante fixed price.

To be more specific the New-Keynesian model alluded to above is based on the existence of a foreign exchange premium. Denoting the logarithm of the nominal exchange rate of a country as the logarithm of the price of the foreign currency in units of the domestic currency (s_t) and letting $i_t$and i_t^* stand for, respectively, the nominal interest rate on a risk-less deposit in the domestic and foreign currency, this implies that we have the following relationship:

$$i_t = i_t^* + E_t(s_{t+1}) - s_t - \lambda_t, \tag{1}$$

where λ_t stands for the foreign exchange rate premium - or excess premium - at time period t and $E_t(\mathrm{s}_{t+1})$ represents the rational expectation of s_{t+1},conditional on all information available to the market at time t. Adding and subtracting the realised and ex ante expected domestic and foreign price level at, respectively, time period t (p_t and p_t^*) and t+1 (p_{t+1} and p_{t+1}^*), and defining the real exchange rate (q_t) by the relationship $q_t = s_t + p_t^* - p_t$, (1) can be rewritten as:

$$i_t - i_t^* = E_t(q_{t+1}) - q_t + E_t(\pi_{t+1} - \pi_{t+1}^*) - \lambda_t, \tag{2}$$

where $\pi_{t+1}$and π_{t+1}^*stands for, respectively, $p_{t+1} - p_t$ and $p_{t+1}^* - p_{t.}^*$

The model version presented here is based on the two countries being fully symmetric and with identical preferences and technology. Hence, there is no "home bias" in consumption and monetary policy is in both countries assumed to set its interest rate endogenously in accordance with a similar type of Taylor rule. In this rule the interest rate - in addition to itself lagged (smoothing) - reacts to the deviation of inflation from the target of the central bank, the output gap and possibly other variables. Denoting the long-run equilibrium interest rates in the two countries, or the "Wicksellian" real interest rates[3], by respectively, $\tilde{r}_t$ and $\tilde{r}_t^*$, the output and inflation gaps by, respectively, $u_t^i - \bar{u}^i$, and $\pi_t^i - \bar{\pi}^i$, where i stands for either a blank and a *,

[2]The exposition in this section is based on the model developed in (9)

[3]The "Wicksellian real interest rate is the real interest that would prevail if prices fully adjusted instantaneously and the economy is in equilibrium all the time. In other words, if the central banks hit their inflation targets at all times, the output gaps are closed and $\varepsilon_t^i = 0$, they would want $i_t^i - \bar{\pi}^i$ to equal $\tilde{r}_t^i$ in 3 and 4, for $i =$ and *, respectively.

and a bar representing, respectively, potential output and the inflation target, such a rule for the domestic and foreign country would take the form:

$$i_t = (1-\alpha)\bar{\pi} + \tilde{r}_t + \sigma(\pi_t - \bar{\pi}) + \rho(u_t - \bar{u}) + \alpha(i_{t-1} - \tilde{r}_{t-1}) + \varepsilon_t \tag{3}$$

and

$$i_t^* = (1-\alpha)\bar{\pi}^* + \tilde{r}_t^* + \sigma(\pi_t^* - \bar{\pi}^*) + \rho(u_t^* - \bar{u}^*) + \alpha(i_{t-1}^* - \tilde{r}_{t-1}^*) + \varepsilon_t^* \ , \tag{4}$$

where the residuals ε_t and ε_t^* accounts for other influences in setting the two interest rates, so that a higher value of, respectively, ε_t and $\varepsilon_{t,}^*$ would mean tighter money, respectively, at home and abroad. In 3 and 4 the Taylor principle is assumed to apply so that $\sigma > 1-\alpha$. Assuming that $\bar{\pi} = \bar{\pi}^*$,$\bar{u} = \bar{u}^*$ and $\tilde{r}_t = \tilde{r}_t^*$, 3 and 4 imply that

$$i_t - i_t^* = \sigma(\pi_t - \pi_t^*) + \rho(u_t - u_t^*) + \alpha(i_{t-1} - i_{t-1}^*) + \varepsilon_t - \varepsilon_t^* \tag{5}$$

Since domestic and foreign consumers have identical preferences, if faced with the same price they would end up demanding the same basket, and purchasing parity would hold all the time. However, in this set-up we assume there is pricing to market that arises from a particular type of nominal price stickiness, denoted local currency pricing, or LCP, in the literature. In this kind of price setting firms are assumed to set different prices for their goods sold in each country and each of those prices is sticky in the currency in which it is set. To incorporate such a mechanism, the model is based on so called Calvo price setting, where each firm at every point of time has a given and constant probability of changing its two prices.

To be more specific this means that the log of the aggregate price of the goods produced at home for sale in the domestic market (denoted by a subscript H) is given by

$$p_{Ht} = (1-\Theta)\tilde{p}_{Ht} + \Theta p_{Ht-1}, \tag{6}$$

where $\tilde{p}_{Ht}$is the log of the price of the fraction of firms $(1-\Theta\ )$ that gets the possibility of changing their price in period t and Θ denotes the constant hazard rate (or probability) of not changing the price.

Firms produces output using only labor, so $w_t - a_t$ is the log of the unit cost, where w_t is the the log of the nominal wage, and a_t the log of labor productivity. With a discount factor equal to β and a probability Θ that the price will not change, the firms that get the opportunity to reset their prices at time t set them to maximise the expected present discounted value of future profits, which means that the optimal price satisfies the following recursion:

$$\tilde{p}_{Ht} = (1-\Theta\beta)(w_t - a_t) + \Theta\beta E_t \tilde{p}_{Ht+1}. \tag{7}$$

With a bit of manipulation[4], equations 6 and 7 can be combined to arrive at

$$\pi_{Ht} = \delta(w_t - a_t - p_{Ht}) + \beta E_t \pi_{Ht+1}, \tag{8}$$

where $\pi_{Ht} = p_{Ht} - p_{Ht-1}$and $\delta = (1-\Theta)(1-\Theta\beta)/\Theta$. By looking at the expression for δ, it is worth noting that the greater the likelihood is that a business will be able to change its prices, that is the lower is Θ, the greater is δ.

[4]Inserting the expression for $\tilde{p}_{Ht}$ in 7 into 6 $\Rightarrow p_{Ht} = (1-\Theta)(1-\Theta\beta)(w_t - a_t) + \Theta\beta(1-\Theta)E_t\tilde{p}_{Ht+1} + \Theta p_{Ht-1}$. Solving 6 for $(1-\Theta)E_t\tilde{p}_{Ht+1}$ and substituting the resulting expression for the similar term in the aforementioned equation and adding and substituting $(1-\Theta)(1-\Theta\beta)p_{Ht}$, should then lead us almost directly to the expression in 8.

Following similar steps as in the derivation of 8 and realizing that the cost per unit in foreign currency terms of the home good is $w_t - s_t - a_t$,we can arrive at an equation for the evolution of prices of home goods sold in the foreign country in terms of foreign currency, p^*_{Ht}:

$$\pi^*_{Ht} = \delta(w_t - a_t - s_t - p^*_{Ht}) + \beta E_t \pi^*_{Ht+1}, \tag{9}$$

where $\pi^*_{Ht} = p^*_{Ht} - p^*_{Ht-1}$. Combining 8 and 9 gives:

$$\pi_{Ht} - \pi^*_{Ht} = \delta(s_t + p^*_{Ht} - p_{Ht}) + \beta E_t(\pi_{Ht+1} - \pi^*_{Ht+1}) \tag{10}$$

So far we have only looked at the price setting of goods produced at home. However, under the assumption of symmetry, we know that the deviations from the law of one price should be equal for home and foreign produced goods. Denoting the price of foreign goods at home and abroad, the last one in terms of foreign currency, by respectively p_{Ft} and p^*_{Ft},we therefore have that:

$$p_{Ht} - s_t - p^*_{Ht} = p_{Ft} - s_t - p^*_{Ft} \Leftrightarrow p_{Ft} - p_{Ht} = p^*_{Ft} - p^*_{Ht} \tag{11}$$

Furthermore, the assumption of identical preferences implies that the aggregate price indices at home and abroad are given by, respectively, $p_t = (p_{Ht} + p_{Ft})/2$ and $p^*_t = (p^*_{Ht} + p^*_{Ft})/2$, which due to 11 implies that $\pi_{Ht} - \pi^*_{Ht} = \pi_{Ft} - \pi^*_{Ft} = \pi_t - \pi^*_t$ and $q_t = s_t + p^*_t - p_t = s_t + p^*_{Ht} - p_{Ht} = s_t + p^*_{Ft} - p_{Ft}$. Based on these identities 10 can be expressed in aggregate terms, implying that

$$\pi_t - \pi^*_t = \delta q_t + \beta E_t(\pi_{t+1} - \pi^*_{t+1}), \tag{12}$$

which is nothing else than the aggregate Phillips curve for our two-country two-markets economy.

Solving 12 for $E_t(\pi_{t+1} - \pi^*_{t+1})$ and inserting the resulting expression for the equivalent expectation term in 2 implies:

$$i_t - i^*_t = E_t(q_{t+1}) - (\frac{\beta + \delta}{\beta})q_t + \frac{1}{\beta}(\pi_t - \pi^*_t) - \lambda_t \tag{13}$$

Equating this expression to 5 and solving for $E_t q_{t+1}$=>

$$E_t(q_{t+1}) = (\frac{\sigma\beta - 1}{\beta})(\pi_t - \pi^*_t) + (\frac{\beta + \delta}{\beta})q_t + \alpha(i_{t-1} - i^*_{t-1}) + \rho(u_t - u^*_t) - \lambda_t + \varepsilon_t - \varepsilon^*_t \tag{14}$$

The real version of the foreign exchange premium relationship, 2, along with the home versus foreign inflation relationship in 12 and the two Taylor rules, 3 and 4, will thus allow us to derive a three-equation dynamic system for the simultaneous determination of home relative to foreign inflation, the real exchange rate, and the domestic versus foreign interest rate differential. This relationship is given by:

$$E_t \begin{pmatrix} \pi_{t+1} - \pi^*_{t+1} \\ q_{t+1} \\ i_t - i^*_t \end{pmatrix} = \begin{bmatrix} \frac{1}{\beta} & -\frac{\delta}{\beta} & 0 \\ \frac{\sigma\beta - 1}{\beta} & \frac{\beta+\delta}{\beta} & \alpha \\ \sigma & 0 & \alpha \end{bmatrix} \begin{pmatrix} \pi_t - \pi^*_t \\ q_t \\ i_{t-1} - i^*_{t-1} \end{pmatrix} + \begin{bmatrix} 0 & 0 \\ -1 & \rho \\ 0 & \rho \end{bmatrix} \begin{pmatrix} \lambda_t \\ u_t - u^*_t \end{pmatrix} + \begin{pmatrix} 0 \\ \varepsilon_t - \varepsilon^*_t \\ \varepsilon_t - \varepsilon^*_t \end{pmatrix} \tag{15}$$

Alternatively this system might be written forwardly as[5] [6]

$$\begin{pmatrix} \pi_t - \pi_t^* \\ q_t \\ i_t - i_t^* \end{pmatrix} = \begin{bmatrix} \beta+\delta & \delta & -\delta \\ 1 & 1 & 1 \\ \frac{1}{\alpha}\sigma(\beta+\delta) & -\frac{1}{\alpha}\sigma\delta & \frac{1}{\alpha}(\sigma\delta+1) \end{bmatrix} E_t \begin{pmatrix} \pi_{t+1} - \pi_{t+1}^* \\ q_{t+1} \\ i_{t+1} - i_{t+1}^* \end{pmatrix}$$
$$+ \begin{bmatrix} \beta+\delta & \delta & -\delta \\ 1 & 1 & 1 \\ \frac{1}{\alpha}\sigma(\beta+\delta) & -\frac{1}{\alpha}\sigma\delta & \frac{1}{\alpha}(\sigma\delta+1) \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -1 & \rho \\ 0 & \rho \end{bmatrix} \begin{pmatrix} \lambda_t \\ u_t - u_t^* \end{pmatrix}$$
$$+ \begin{bmatrix} \beta+\delta & \delta & -\delta \\ 1 & 1 & 1 \\ \frac{1}{\alpha}\sigma(\beta+\delta) & -\frac{1}{\alpha}\sigma\delta & \frac{1}{\alpha}(\sigma\delta+1) \end{bmatrix} \begin{pmatrix} \omega_{\pi(t+1)} \\ \omega_{q(t+1)} \\ \varepsilon_t - \varepsilon_t^* \end{pmatrix}, \quad (16)$$

where $\omega_{\pi(t+1)} = E_t(\pi_{t+1} - \pi_{t+1}^*) - (\pi_{t+1} - \pi_{t+1}^*)$ and $w_{q(t+1)} = E_t(q_{t+1}) - q_{t+1}$.At time t, only one of the variables is predetermined as both $\pi_t - \pi_t^*$ and q_t may jump in response to contemporaneous domestic and foreign shocks. Algebraically, for the dynamic system to have a unique stable solution, two roots of the matrix in front of the time $t+1$ dated variables in 16 must be forward stable in the sense of being less than one (or inside the unit circle), which requires that $\sigma + \alpha > 1$. This is nothing but the Taylor condition referred to earlier and implies that the two central banks ultimately raise nominal interest rates more than one-for one with an increase in inflation.

To substantiate what is said in the preceding paragraph we can simplify matters by considering a special case where the errors to the monetary rules are *i.i.d.* random variables with mean zero. To allow for a simpler algebraic solution we will also assume that $\sigma + \alpha = 1/\beta$, a restriction that at the same time guarantees that the Taylor principle is fulfilled by assumption. Under this restriction on the parameters, the eigenvalues of the matrix in front of the time $t+1$ dated variables in 16 are given by β, $\frac{1}{\mu_1}$ and $\frac{1}{\mu_2}$, where

$$\mu_1 = \frac{1}{2}(1 + \alpha + \frac{\delta}{\beta} + \sqrt{(1 + \alpha + \frac{\delta}{\beta})^2 - 4\alpha}) > 1$$

and $\mu_2 = \frac{\alpha}{\mu_1} < 1$. Hence, two of the roots of the matrix in front of the time $t+1$ dated variables in 16 are in this case demonstrated to be forward stable while one root is forward unstable, a prerequisite for the existence of a uniquely determined (or determinate) equilibria in the case where two of the endogenous variables are free and one is characterised as a predetermined variable. However, in this context it is important to stress that if this was not the case and for instance only one of the roots were forward stable in the sense of being less than one in absolute value, we would be in a situation where the number of free variables are greater than the number of forward stable roots, and thus a solution characterised by the existence of multiple equilibria and self-fulfilling prophesies. All roots being forward stable would on the other hand mean that the system is generically unstable and thus has no solution.

In the case where the solution of the system in 16 turns out to be unambiguous, the solution is in the general case somewhat complicated. However, in the case where the endogenous variable vector, y_t, can be partitioned into two separate vectors consisting, respectively, of the

[5] $$\begin{bmatrix} \frac{1}{\beta} & -\frac{\delta}{\beta} & 0 \\ \frac{\sigma\beta-1}{\beta} & \frac{\beta+\delta}{\beta} & \alpha \\ \sigma & 0 & \alpha \end{bmatrix}^{-1} = \begin{bmatrix} \beta+\delta & \delta & -\delta \\ 1 & 1 & 1 \\ \frac{1}{\alpha}\sigma(\beta+\delta) & -\frac{1}{\alpha}\sigma\delta & \frac{1}{\alpha}(\sigma\delta+1) \end{bmatrix}$$

[6] I used the the word forward to emphasize that the equations that we will study describe how combinations of the state variables depend on expectations of the future rather than how they depend on the past. The reversion of the direction in which the equations are presented - and alas solved - also reverses the conditions under which an equation is either stable or unstable.

free variables, y_t^1, and the predetermined variables, y_t^2, and the system is contingent on a set of other variables forming an exogenous forcing variable vector, here denoted by f_t, the solution is given in the appendix by the two expressions in equations 35 and 39 combined. Going forward, however, we do not intend to continue calculating these solutions by hand, but to use suitable software developed for such a purpose and to refer to the explicit formula if and only if that is appropriate in the individual case.[7]

3 Data

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4 Results

In implementing the system in KVARTS, we have chosen to use two different strategies: i) one that is based on an freely estimated SVECM model, but where the expectations are required to be adaptive in the sense that expectations are formed on the basis of estimated backward looking autoregressive processes or VAR models and ii) one where we operate with a fully model consistent rational expectation formation where the solution is required to be in accordance with the saddle path of a calibrated version of the previously discussed underlying theoretical structure.

This approach is in many ways consistent with the one underlying the so-called FRB/US model in the United States ((3)) and would allow users of the model to switch between alternative assumptions regarding expectation formations when running KVARTS. In principle, it would be possible to run the model on the basis of both a purely backward-looking and forward-looking expectation structure. In the case of forward-looking expectations, the model could be run in two modus operandi: one where expectations are adaptive according to an estimated backward-looking AR process of inflation, alternatively a small stylized reduced form representation where the rest of the economy is modelled according to a small stylized VAR model, and one in which expectation is consistent with a New Keynesian rational expectation model.

4.1 Estimation and implementation of a Structural Vector Equilibrium Correction Model (SVECM) with forward looking expectations

Based on the theoretical framework outlined in Section 2, we have attempted to design and estimate an empirical Structural Vector Equilibrium Correction Model (SVECM) for the simultaneous determination of the nominal euro-krone exchange rate, the money market interest rate and prices. The starting point for this modelling has been a general dynamic structural model set-up that encompasses the theoretical general equilibrium model outlined in Section 2 and as specified in system 15 and 16.[8] By referring to the variables contained in 15 and defining the

[7] To solve the log-linearized versions of the DSGE models presented in this paper we have utilized the DYNARE package (10). This package is based on the Method of Blanchard and Kahn (1). For an exposition of this method we refer to the appendix of this paper.

[8] The approach taken here is based on the idea of preserving the long-run properties of the theoretical model while at the same time being flexible in the dynamic design of the model. This has been done to increase the model's ability to survive a confrontation with the data, though at the potential cost of jeopardizing the models' systemic properties. Interestingly in this respect, in (6) the opposite approach is proposed and taken. That is one

two vectors $\bar{Z}$ and $\bar{X}$ by, respectively

$$\bar{Z} = (p_t, p_t^*, i_t, i_t^*, \lambda_t, u_t, u_t^*, \Delta p_t, \Delta p_t^*)$$

and

$$\bar{X} = (i_t^*, \lambda_t, u_t, u_t^*, \Delta p_t, \Delta p_t^*)$$

this general point of departure can be given the following unrestricted general representation:[9]

$$\begin{aligned}\Delta s_t = & -\phi_s\{(s_{t-1} + p_{t-1}^* - p_{t-1}) + \alpha_s(i_{t-1} - i_{t-1}^*) - \omega_s(\Delta p_{t-1} - \Delta p_{t-1}^*) \\ & - \gamma_s(u_{t-1} - u_{t-1}^*)\} + c_s + \lambda_{t-1} + \sum_{i=0}^{k} \Gamma_i \Delta \bar{Z}_{t-i} + \sum_{i=1}^{k} \rho_i \Delta s_{t-i} + \Theta_s D_t + \epsilon_t^s\end{aligned}$$

$$\begin{aligned}\Delta i_t = & -(\alpha_i)\{(i_{t-1} - i_{t-1}^*) - \psi_i(\Delta p_{t-1} - \Delta\ p_{t-1}^*) + \gamma_i(u_{t-1} - u_{t-1}^*)\} \\ & + c_i + \sum_{i=0}^{k} \Upsilon_i \Delta \bar{X}_{t-i} + \sum_{i=1}^{k} \psi_i \Delta i_{t-i} + \Theta_i D_t + \epsilon_t^i \quad (17) \\ \Delta p_t = & \Delta p_t^* + c_\pi + \delta(p_{t-1} - (s_{t-1} + p_{t-1}^*)) + \beta E_t(\Delta p_{t+1} - \Delta p_{t+1}^*) + \epsilon_t^p\end{aligned}$$

Substituting all theory variables by their empirical equivalents[10] and using 17 as the starting point for a structural design or reduction process, we end up with the following parsimonious

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aims at preserving the model's dynamic and systemic properties while being flexible in the design of the models long-run structure.

[9]In System 17 more or less all the restrictions imposed on 15 have been undone and π and π^* substituted by, respectively Δp and Δp^*, where Δ represents the first difference of the variables and small letters symbolize logarithmic transformations. The parameters ϕ_s,$\alpha_i$and δ are the loadings or error correction parameters pertaining to the dynamic equations of, respectively, the exchange rate, domestic interest rate and the domestic price index, while the parameters α_s, ω_s, ω_i, $\gamma_s$and γ_i pertain to the long-run cointegration structure of the exchange rate (subscript s) and the interest rate equation (subscript i), respectively. The two matrices Γ_i and Υ_i are assumed to be conformable and to contain the parameters pertaining to extrinsic non-eigen dynamic effects in, respectively, the exchange rate and interest rate equation, while ρ_i and ψ_i capture the parametrical representation of the two equation's eigen-dynamics. D_t represents deterministic terms like centered seasonals, step and blip dummies etc.. Note that in 17 the foreign exchange premium $\lambda_{\{t\}}$ has not been modelled as a parametrized function of variables meant to capture such an effect. In 17 we have also specified three equation-contingent error terms, ϵ_t^j , where j is assumed to take on the alpha numeric letters of s,i and p, respectively,to indicate to which equation it belongs.

[10]When it comes to the Phillips curve, we have chosen to model it based on the Norwegian concept of core inflation, in other words inflation adjusted for energy and taxes. Core inflation is also used as an inflation proxy in the interest rate rule, where the Norwegian money market rate in addition to the money market rate for the euro area, is assumed to depend on, respectively, the deviation of the rate of unemployment and core inflation from its equilibrium level and inflation target. As far as the Euro-krone exchange rate is concerned, to intercept the effect of price changes we have tried out both using the consumer price and the core index, the first one necessitating an extension of the model to cope with two different but related concepts of price movements. The foreign exchange risk premium on the other hand is meant to be intercepted by the oil price (*poil*) in the short run and the ratio of the value of gas and oil exports to the value of total exports (*vogava*) in the long run.

structural representation.

$$
\begin{aligned}
\Delta s_{t-1} = \underset{(0.04)}{0.09} + \underset{(0.089)}{0.229}\Delta s_{t-2} + \underset{(0.003)}{0.007}\Delta(p_t - p^*_t) - \underset{(0.001)}{0.003}\Delta(p_{t-1} - p^*_{t-1}) \\
- \underset{(0.001)}{0.004}\Delta(p_{t-2} - p^*_{t-2}) - \underset{(0.013)}{0.067}\Delta poil_t - \underset{(0.001)}{0.004}\Delta(i_t - i^*_t) \\
- \underset{(0.056)}{0.174}(s_{t-1} + p^*_{t-1} - p_t) - \underset{(0.0007)}{0.0019}((i_{t-1} - \Delta p_{t-1}) - (i^*_{t-1} - \Delta p^*_{t-1})) \\
- \underset{(0.021)}{0.054}(vogava_{t-1}) + \underset{(0.056)}{0.174}\,\text{SDUM081084} + \epsilon^s_t
\end{aligned}
$$

$$
\begin{aligned}
\Delta i_t = \underset{(0.11)}{0.168} + \underset{(0.05)}{0.97}\Delta i^*_t + \underset{(0.035)}{0.134}\Delta i_{t-1} + \underset{(0.038)}{0.147}\Delta\Delta_4(p_t) \\
- \underset{(0.023)}{0.083}(i_{t-1} - i^*_{t-1}) - \underset{(0.028)}{0.074}u_{t-1} + \underset{(0.033)}{0.203}\Delta_4 p_{t-1} \\
- \underset{(0.095)}{0.519}\,\text{SDUM031032}) - \underset{(0.12)}{1.09}(\text{DUM033}) - \underset{(0.12)}{0.39}(\text{DUM041}) \\
- \underset{(0.075)}{0.398}\,\text{SDUM082093} - \underset{(0.049)}{0.247}\,\text{SDUM133} - \underset{(0.049)}{0.247}\,\text{CS1} + \epsilon^i_t
\end{aligned}
$$

$$
\Delta p_t = -\underset{(2.96)}{6.51} + \Delta p^*_t - \underset{(4.08)}{8.98}(p_{t-1} - (s_{t-1} + p^*_{t-1})) + \underset{(0.137)}{0.534}\Delta_2(p_{t+2} - p^*_{t+2})
$$

Vector SEM-AR 1-1 test:F(9,126) $=$ 1.4132[0.1892]

Vector Normality test: $\chi^2(6) = 7.9898$ [0.2389]

In 4.1 we have modelled the three-dimensional dynamic structure of the logarithm of the krone-euro exchange rate (s), the three-month money market interest rate (i), and the logarithm of the core price index (that is the consumer price index adjusted for taxes and energy prices) using quarterly data from the first quarter of 2001 until the second quarter of 2017. As will become clear when we later take a closer look at the system's long-term structure below, the estimated SVECM model encompasses both the exchange rate equation and the Taylor rule in 15. More surprising in this respect is perhaps the fact that the system contains and thus provides support for the existence of a Phillips curve. Importantly, this is a finding that is fully consistent with a New-Keynesian rational expectation equilibrium framework, and which forms the basis for forward-looking expectations in the process governing the simultaneous determination of the exchange rate, prices and interest rates *per se*. A superficial look at the system in 4.1 does, however, make clear that the dynamics deviate significantly from the one that follows from the New-Keynesian optimization problem per se and it has been necessary to equip the model with a number of deterministic variables such as dummies for structural breaks and seasonality to cope with the worse instances of unexplained systemic information. It should also be mentioned that the estimated persistence in the exchange rate equation of our SVECM model is not consistent

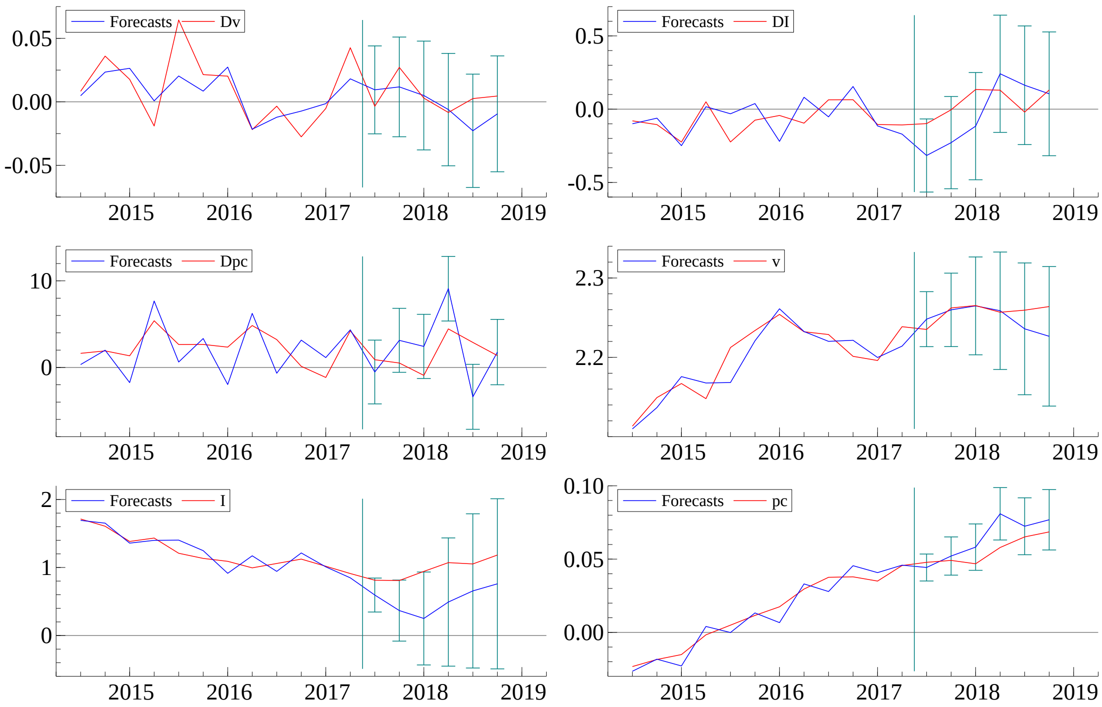

FIGURE 1: Dynamic Forecasts

with that which follows from theory, in the sense that it implies a stable backward-looking exchange rate process and not one that would guarantees a unique stable saddle path solution in a three dimensional structure with two free variables.[11]

Looking at the system diagnostics and stability tests in Figure 8 , the system seems to be fairly well specified. Extending the data set until out 2018 and using the model to simultaneously and dynamically forecast 6 period ahead, that is without conditioning the forecasts of the endogenous variables on anything other than exogenous processes and without restricting the lagged observations of the endogenous variables to be in accordance with history, but to follow their earlier forecasted values, shows that the forecasts lie within their forecast confidence intervals with one exception; that is the inflation and price level, both laying below their estimated confidence intervals in the second quarter of 2018. Otherwise, and given the fact that the model implies an interest rate trajectory that lies substantially below the realised one for the whole forecast period starting in the third quarter 2017, we notice that the model does not seem to explain the krone exchange rate particularly well, in particular during the second last part of 2018 where the krone-euro exchange rate followed a much weaker trajectory than the one forecast by the model.

By letting the logarithm of the value of oil and gas as a share of the total export value

[11] Taken at face value, one would perhaps have though that this should imply the existence of multiple equilibria and self-fulfilling prophecies, but simulations of the model based on a Gauss-Seidel iterative scheme across all the observations of the sample and imposing the same kind of parametrical restriction on a fully consistent version of the theoretical rational expectation model, is suggestive of explosive roots and instability.

($vogava$) intercept the effect of a long-run foreign exchange premium and writing the rate of unemployment and inflation as deviations from their target equivalents, the long-run structure of the estimated SVECM in 4.1 is given by:

$$s_t = p_t - p_t^* - 0.11((i_t - \Delta p_t) - (i_t^* - \Delta p_t^*)) - 0.31(vogava_t)$$

$$i = i^* - 0.89(u - \bar{u}) + 2.45(\Delta_4 p - \pi) \tag{18}$$

$$p_t = s_t + p_t^*$$

According to the long-run cointegration structure in 18 the (logarithm of) the real exchange rate is a stationary variable. Given this, the long run relationship governing the nominal euro-krone exchange rate implies either that the real interest rate differential vis-a-vis abroad, $((i_t - \Delta p_t) - (i_t^* - \Delta p_t^*))$, cointegrates with the long-run foreign exchange rate premium proxy of $vogava$, or that the two last mentioned variables are stationary per se. Beyond that and the fact that the equation implies a full exchange rate pass through of relative price changes in the long run, the equation implies that a one percentage point's increase in the real interest rate differential vis-a-vis abroad and a one percent increase in the value of the oil and gas exports as a proportion of the value of total exports is estimated to lead to a weakening of the crone exchange rate of, respectively, 0.11 and 0.31 percent. Comparing the structure in 18 with the theoretical structure in 15, we se also that the Taylor principle is fulfilled in the long run, a one percentage point increase in inflation relative to its target in fact leading to an increase in the nominal interest rate of 2,45 percentage points. The long-run interest rule equation also implies that a one percentage point increase in the rate of unemployment (relative to its trend level) will lead to a long-run interest rate decline of 0.89 percentage points. Otherwise a one percentage point rise in the foreign interest rate is imposed to lead to an equivalent rise in the domestic interest rate in the long run, according to the dynamic specification of the policy rule in 4.1, not far from its short run response of 0.97.

4.2 Calibration and Implementation of a model with fully model consistent expectations

$$q_t = s_t + p_t^* - p_t \tag{19}$$

$$\pi_{t+1} - \pi_{t+1}^* = \frac{1}{\beta}\Delta(\pi_t - \pi_t^*) + \frac{(-\delta)}{\beta} q_t \tag{20}$$

$$q_{t+1 = (\frac{\beta\sigma - 1}{\beta})\Delta(p_t - p_t^*) + (\frac{\beta+\delta}{\beta}) q_t + \alpha(i_{t-1} - i_{t-1}^*) + \epsilon_t} \tag{21}$$

$$(i - i^*)_t = \alpha(i - i^*)_{t-1} + \sigma(\pi_t - \pi_t^*) + \epsilon_t \tag{22}$$

$$\epsilon_t = \rho\epsilon_{t-1} + \sigma_\epsilon v_t \tag{23}$$

$$\delta = \frac{(1 - \Theta)(1 - \Theta\beta)}{\Theta} \tag{24}$$

In the last subsection a SVECM model for the simultaneous determination of the euro-krone exchange rate, money market interest rate and prices were incorporated in KVARTS by assuming an adaptive expectation formation scheme. This was done to avoid issues related to stability when dealing with model consistent/forward looking expectations in a setting that is poorly suited for such a purpose due to cumbersome dynamics and a structure that does not necessarily underpin the existence of a stable saddle path solution. To be able to incorporate fully model-consistent and rational forward looking expectations in KVARTS we have therefore had to go back to our theoretical point of departure and implement the solution of a saddle-path stable parameterisation of a New-Keynesian structural equilibrium model developed along the lines of the theoretical partial equilbrium model in Section 2 and as reproduced in equations 19 to 24 above. In doing so we have deliberately chosen to simplify matters somewhat compared to the more general set-up in section 2 by considering a model version based upon uncovered interest parity and a policy rule of strict inflation targeting, the implication of which is the full cancellation of the exogenous forcing term in 16. In light of the theoretical framework we have used as a starting point for the construction of the empirical SVECM model of the previous section, this may seem somewhat unsatisfactory. However, it has so far proved more difficult than we had previously thought to implement dynamic structures for the two variables of the forcing variable vector, respectively, the foreign exchange risk premium and the relative unemployment rate, without this having major unwanted consequences for the model's dynamic properties. Another and potentially more serious problem in this context is the fact that the model that we so far have used as a point of reference for our modeling is partial in the sense that it does not include all aspects of the real economy in KVART. In the event that one wishes to extend the model to apply to a monetary policy regime with a flexible inflation-target, and thus to introduce the output gap as an additional explanatory variable in the policy rules of the central banks, as prepared for in Section 2 , this would exclude an potentially important stabilizing feedback mechanism in the model. This will especially apply when implementing the DSGE model as a sub-model in a larger empirical macro econometric model like KVARTS, as such an implementation necessarily will have to be based on an unidirectional line of communication between the partial DSGE model governing the exchange rate and the rest of the model to ensure stability. To be able to take such a feedback mechanism properly into account one would therefore have to leave the more partial equilibrium framework developed in Section 2 to the benefit of a more comprehensive general equilibrium model of the whole economy. As this would require a significant extension of the mandate of this article, if not justifying an entirely independent paper in itself, we have chosen to leave such an extension to future research.

To study the properties of the system above we have calibrated the model by resorting mainly to two different strategies. One, by basing the calibration of the price-rigidity parameter, Θ, the discount factor, β, and the interest rate smoothing parameter, α, on the empirical estimates of the long-run structure of the SVECM model estimated in the last section. And another were we calibrate the model using "counterfactual"[12] parameter values for these parameters based on the values in, respectively, (7) and (8).[13][14] In both cases the other parameters related to the system in 19 to 24 above have been calibrated by using standard values from the

[12] In this context the word countefactual refers to values not being based on the estimates of the SVECM model.

[13] To be more precise we have in this context taken the parameter values for θ and β from the first of these cited sources while the α parameter is taken from the last one,

[14] To test for the sensitivity of the impulse responses related to the degree of price rigidity and the discount factor in the empirically based SVECM model, we have also looked at calibrated model versions where these two sources of calibration to a certain degree have been combined. The results of these additional analyzes will be commented on in the following as long as they contribute to qualify the impulse responses to any significant degree. In this context, it is also important to point out that since we impose the parameter restriction $\sigma + \alpha = 1/\beta$, the contemporary inflation response from the central bank, σ, will vary between simulations.

Parameter	Description	Sim 1	Sim 2	Sim 3	Sim 4
β	Discount factor	0.53	0.99	0.53	0.990
θ	Calvo parameter	0.0873	0.0873	0.75	0.75
σ	Taylor rule inflation response $\sigma = (1/\beta - \alpha)$	0.967	0.090	0.967	0.210
α	Taylor rule degree of interest rate smoothing	0.92	0.92	0.92	0.800
ρ_ϵ	Persistence of monetary policy innovations	0.000	0.000	0.000	0.000
σ_ϵ	Standard Deviation of monetary policy innovations	0.0025	0.0025	0.0025	0.0025

Table 1: Calibration Schemes

literature. In the table below these two alternative ways to calibrate the model are represented by the parameter values in the two outer columns, denoted by, respectively, Sim1 and Sim4, while the two columns in the middle refer to two modifications made with respect to the values in the first column. Respectively, one where we have substituted the empirical estimate of the discount factor with the one typically found in the literature, and another where we have done a similar substitution with respect to the price rigidity parameter.

The impulse responses in the two figures below relate to the effect of a one-period-one-standard-deviation monetary policy shock in the two main calibration alternatives referred to above. As clearly born out by these figures, whether we calibrate our model using the estimated values from our empirical SVECM model or use the values typically referred to in the literature, will have a large and significant implication for the dynamic response paths to a monetary policy shock. Evidently, utilizing counterfactual values lead to a much more protracted response trajectory than resorting to the values backed out from the empirical estimates of our SVECM analysis. The impulse responses related to the two modified versions of the SVECM-based calibration scheme reveal that this is mainly due to the degree of price stickiness, as modifying the discount factor has little to say for the degree of persistence. The combination of a high discount factor and a low degree of price stickiness does, however, lead to a forceful inflation response compared to when the opposite is the case and a larger share of the adjustment will have to take place via interest rates and the exchange rate.

Given our general equilibrium model for the simultaneous determination of the euro krone exchange rate, the money market interest rate and core inflation, time has now come to put the calibrated version of our general equilibrium model into a larger macroeconomic context, that is to implement it into the macro econometric model KVARTS. To assure stability this has been done by forcing the solutions related to each separate calibration scheme to be in accordance with the model's uniquely defined saddle path solution and to avoid any interference that in some way or another is extrinsic to the partial system determining the exchange rate.[15]These saddle path solutions are for the four different calibration schemes given in Table XY and implies that the endogenous variables are linear combinations of the lagged relative nominal interest rate gap and the innovation to the monetary policy shock.

Simulations using KVARTS suggest that when implementing forward-looking expectations in a model that is fully consistent with a New Keynesian rational expectation model, responses to a monetary policy shock will inherit some of the properties applying to the DSGE model per se. In particular they will depend on the degree of price rigidity. in the sense that the higher it is the more protracted become the impulse responses related to a monetary policy shock.

[15]To avoid a problem of over determinedness the rate of inflation has been locally defined when inserting the partial DSGE model in KVARTS, implying that variables that are endogenous to both blocks will be exclusively determined by the outer model of KVARTS.

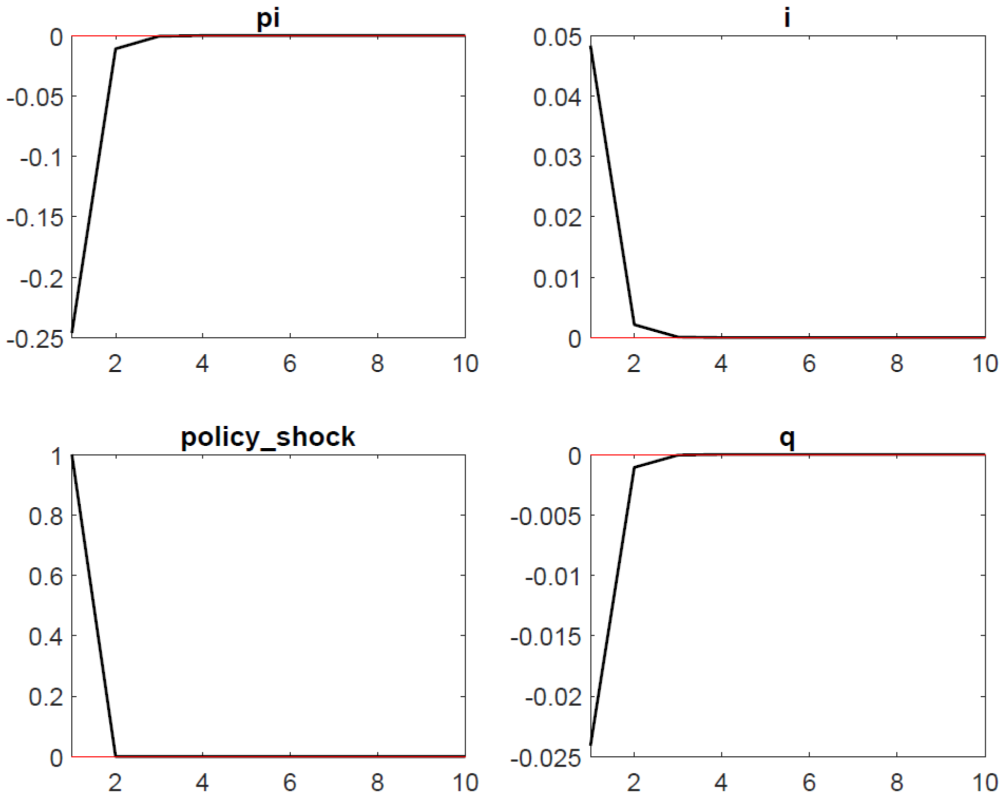

FIGURE 2: Impulse responses: Sim1

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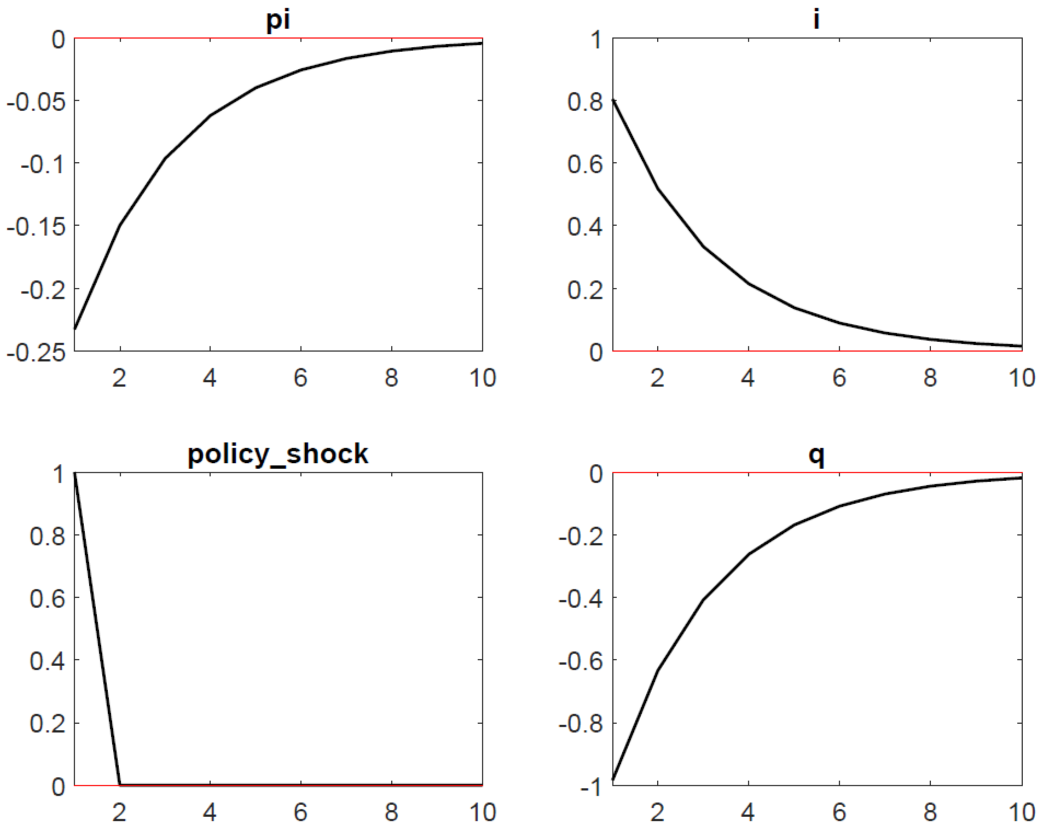

FIGURE 3: Impulse responses: Sim4

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Simulation 1: Empirical parameter values

	pi	i	policy_shock	q
ri_gap(-1)	-90.560436	17.787417	0	-8.868306
eta_eps	-0.246088	0.048335	1.000000	-0.024099

Simulation 2: Empirical parameters, but higher discount factor

	pi	i	policy_shock	q
ri_gap(-1)	-932.190928	32.034623	0	-89.861301
eta_eps	-2.533128	0.087051	1.000000	-0.244188

Simulation 3: Empirical parameters, but higher price stickiness

	pi	i	policy_shock	q
ri_gap(-1)	-41.787037	206.402430	0	-151.165065
eta_eps	-0.113552	0.560876	1.000000	-0.410775

Simulation 4: "Textbook" discount factor and price stickiness, empirical value for interest smoothing

	pi	i	policy_shock	q
ri_gap(-1)	-74.487518	257.400389	0	-314.959870
eta_eps	-0.232773	0.804376	1.000000	-0.984250

TABLE 2: Saddle paths

[10] Stéephane Adjemian, Houtan Bastani, Michel Juillard, Freédeéric Karamé, Junior Maih, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Seébastien Villemot (2011), ŞDynare: Reference Manual, Version 4,Ť Dynare Working Papers, 1, CEPREMAP

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Appendix

Method of Blanchard and Kahn

We are using the Method of Blanchard and Kahn (1) to solve the system in 16. Denoting the three matrices on the right hand side of 16 by respectively A, B and C and partitioning the matrices in accordance with the partitioning of the endogenous variable vector $y_t = (\pi_t - \pi_t^*, q_t, i_t - i_t^*)'$ into two separate vectors $y_t^1 = (\pi_t - \pi_t^*, q_t)'$ and $y_t^2 = (i_t - i_t^*)$, consisting, respectively, of the free variables and the predetermined variables (here only one), 16 can be written as:

$$\begin{pmatrix} y_t^1 \\ y_t^2 \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{pmatrix} y_{t+1}^1 \\ y_{t+1}^2 \end{pmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} f_t + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{pmatrix} \omega_{t+1} \\ \upsilon_{t+1} \end{pmatrix} \tag{25}$$

In 25 $f_t = (\lambda_t,\ u_t - u_t^*)$' represents the set of exogenous forcing variables in 16 and ω_{t+1} and υ_{t+1} stands for, respectively, $(\omega_{\pi(t+1)}, \omega_{q(t+1)})'$ and $\varepsilon_t - \varepsilon_t^*$. The Blanchard and Kahn method begins with a Jordan decomposition of the A matrix, yielding $A = Q\Lambda Q^{-1}$where the diagonal elements of Λ, consisting of the eigenvalues of A, are ordered in increasing absolute value in moving from left to right, and Q stands for the matrix containing the respective eigenvalues' eigenvectors column-wise. Thus the matrix Λ may be written as

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \tag{26}$$

where the eigenvalues in $\Lambda_1$lie within the unit circle, and those in Λ_2 lie outside the unit circle. The eigenvalues in $\Lambda_1$are said to be forward stable since Λ_1^n converges to zero as n approaches

FIGURE 4: KVARTS Impulsresponses of a monetary policy shock with a DSGE modell included in KVARTS using the calibrated values of SIM1 => Low price rigidity.

infinity. The eigenvalues in Λ_2 on the other hand, are said to be unstabløe or explosive, since Λ_2^ndiverges as n diverges. The matrix Q is partitioned conformably as

$$Q = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \tag{27}$$

, where again Q_{11} is conformable with Λ_1, and so forth. If the number of forward stable eigenvalues is equal to the number of non-predetermined variables, which is assumed here, the system is said to be saddle path stable and a unique solution to the model exists. Otherwise, the model is either undetermined, in the sense of generating an infinite number of solutions, or unstable.

Proceeding under the case of saddle-path stability, inserting 26 and 27 in the Jordan decomposed version of 25, and multiplying the system with $Q^{-1}$and denoting the transformed variable vectors by $\widetilde{y}_t^1$ and $\widetilde{y}_t^2$, such that

$$\begin{pmatrix} \widetilde{y}_t^1 \\ \widetilde{y}_t^2 \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix}^{-1} \begin{pmatrix} y_t^1 \\ y_t^2 \end{pmatrix} = \begin{bmatrix} \widetilde{Q}_{11} & \widetilde{Q}_{12} \\ \widetilde{Q}_{21} & \widetilde{Q}_{22} \end{bmatrix} \begin{pmatrix} y_t^1 \\ y_t^2 \end{pmatrix}, \tag{28}$$

we have that

$$\begin{pmatrix} \widetilde{y}_t^1 \\ \widetilde{y}_t^2 \end{pmatrix} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix} \begin{pmatrix} \widetilde{y}_{t+1}^1 \\ \widetilde{y}_{t+1}^2 \end{pmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} f_t + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{pmatrix} \omega_{t+1} \\ \upsilon_{t+1} \end{pmatrix} \tag{29}$$

, where

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \widetilde{Q}_{11} & \widetilde{Q}_{12} \\ \widetilde{Q}_{21} & \widetilde{Q}_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{30}$$

FIGURE 5: KVARTS Impulsresponses of a monetary policy shock with a DSGE modell included in KVARTS using the calibrated values of SIM4 => High price rigidity.

and

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} = \begin{bmatrix} \widetilde{Q}_{11} & \widetilde{Q}_{12} \\ \widetilde{Q}_{21} & \widetilde{Q}_{22} \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \tag{31}$$

This transformation effectively "decouples" the system, so that the non-predetermined variables depend only upon the forward stable eigenvalues of the A matrix contained in Λ_1 as expressed in the upper part of 29.

Having decoupled the system, we derive a solution for the non-predetermined variables by taking the expectation on both sides of 29 (remember $E_t(\omega_{t+1}) = E_t(\upsilon_{t+1}) = 0$) and performing a forward iteration on the upper portion of the equation system. This is accomplished as follows. First, re-express, the upper portion of 29 as

$$\widetilde{y}_t^1 = \Lambda_1 E_t(\widetilde{y}_{t+1}^1) + F_1 f_t \tag{32}$$

This implies an expression for $\widetilde{y}_{t+1}^1$ of the form

$$\widetilde{y}_{t+1}^1 = \Lambda_1 E_t(\widetilde{y}_{t+2}^1) + F_1 E_t f_{t+1} \tag{33}$$

which can be substituted into 32 to obtain

$$\widetilde{y}_t^1 = \Lambda_1^2 E_t(\widetilde{y}_{t+2}^1) + \Lambda_1 F_1 E_t f_{t+1} + F_1 f_t \tag{34}$$

Since Λ_1 contains stable eigenvalues, Λ_1^n disappears as n approaches infinity. Continuation of this process of iteration thus gives

$$\widetilde{y}_t^1 = \sum_{i=0}^{\infty} \Lambda_1^i F_1 E_t(f_{t+i}) \tag{35}$$

By using 28, we can map this expression back into an expression for y_t^1. We then get

$$\widetilde{y}_t^1 = \widetilde{Q}_{11} y_t^1 + \widetilde{Q}_{12} y_t^2 \Rightarrow y_t^1 = \widetilde{Q}_{11}^{-1} \widetilde{y}_t^1 - \widetilde{Q}_{11}^{-1} \widetilde{Q}_{12} y_t^2.$$

Inserting the expression for $\widetilde{y}_t^1$ in 35 in the above expression gives then

$$y_t^1 = \widetilde{Q}_{11}^{-1} \sum_{i=0}^{\infty} \Lambda_1^i F_1 E_t(f_{t+i}) - \widetilde{Q}_{11}^{-1} \widetilde{Q}_{12}^i y_t^2. \tag{36}$$

Finally, to solve the nonexplosive portion of the system, we can start by multiplying 18 by A^{-1} and setting the disturbance term equal to its zero-expected value, giving

$$\begin{pmatrix} y_{t+1}^1 \\ y_{t+1}^2 \end{pmatrix} = \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix} \begin{pmatrix} y_t^1 \\ y_t^2 \end{pmatrix} + \begin{bmatrix} \widetilde{A}_{11} & \widetilde{A}_{12} \\ \widetilde{A}_{21} & \widetilde{A}_{22} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} f_t, \tag{37}$$

where the $\widetilde{A}_{ij}$ matrices represent the partitions of A^{-1} conformable with y_t^1 and y_t^2. By expanding the lower part of 37 we then get that

$$y_{t+1}^2 = \widetilde{A}_{21} y_t^1 + \widetilde{A}_{22} y_t^2 + (\widetilde{A}_{21} B_1 + \widetilde{A}_{22} B_2) f_t \tag{38}$$

Substituting for y_t^1 using 36 then yields a solution for y_t^2depending on the exogenous forcing variables and the past.

$$y_{t+1}^2 = \widetilde{A}_{21} \widetilde{Q}_{11}^{-1} \sum_{i=0}^{\infty} \Lambda_1^i F_1 E_t(f_{t+i}) + \widetilde{(A}_{22} - \widetilde{A}_{21} \widetilde{Q}_{11}^{-1} \widetilde{Q}_{12}^i) y_t^2 + (\widetilde{A}_{21} B_1 + \widetilde{A}_{22} B_2) f_t \tag{39}$$

Additional

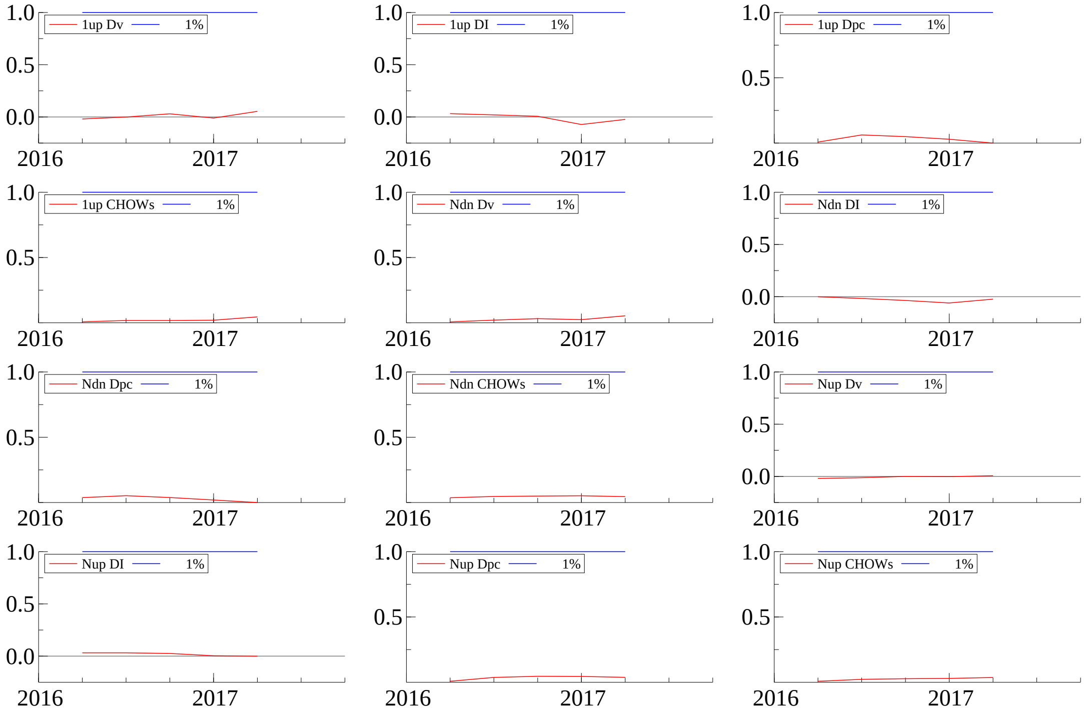

FIGURE 6: System stability tests.

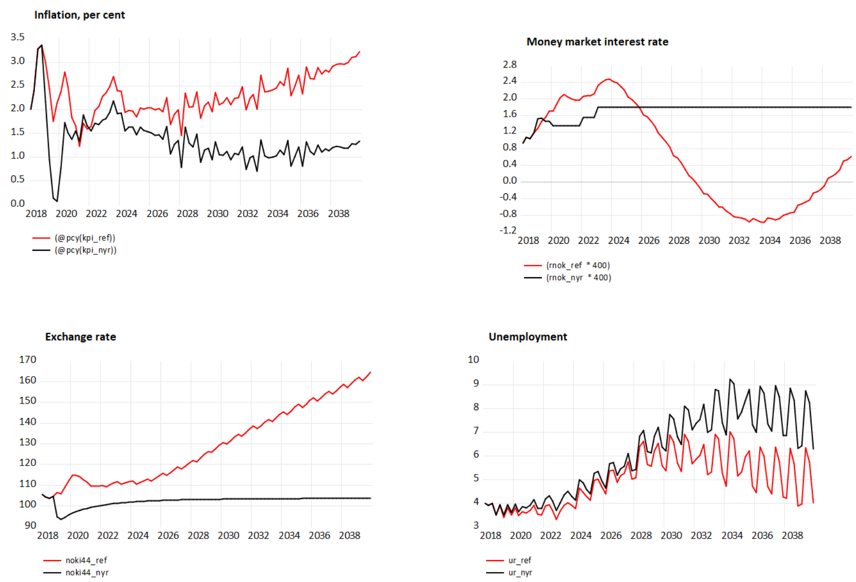

FIGURE 7: KVARTS simulations with (nyr) and without (ref) DSGE model. Calibration SIM1=> Low perice rigidity

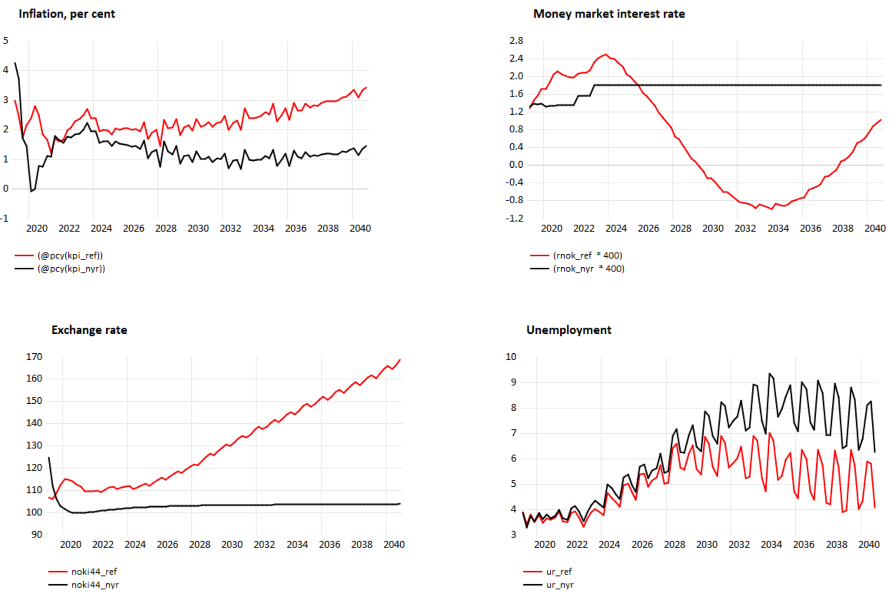

FIGURE 8: KVARTS simulations with (nyr) and without (ref) DSGE model. Calibration SIM4=> High price rigidity